Lecture 12, Oct 17, 2023

Kalman Filter (Discretization) Example

- Consider a system modelled by $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{u}{m}$, where the state is $\boldsymbol{x} = \begin{bmatrix} x \\ v \end{bmatrix}$
- First we need to discretize the system and bring it into standard form $\boldsymbol{x}_{k+1} = \boldsymbol{A}_k\boldsymbol{x}_k + \boldsymbol{B}_k\boldsymbol{u}_k + \boldsymbol{s}_k$ and $\boldsymbol{z}_k = \boldsymbol{D}_k\boldsymbol{x}_k + \boldsymbol{w}_k$
- $\frac{x_{k+1} - x_k}{\Delta t} = v_k + r_k, \frac{v_{k+1} - v_k}{\Delta t} = \frac{u_k}{m} + s_k$ where r_k and s_k are noise terms
  - This gives $x_{k+1} = x_k + v_k\Delta t + r_k\Delta t, v_{k+1} = v_k + \frac{\Delta t}{m}u_k + s_k\Delta t$
- Take some timestep Δt, then $\begin{bmatrix} x_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_k \\ v_k \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta t/m \end{bmatrix} u_k + \Delta t\boldsymbol{s}_k$ and $z_k = v_k = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x_k \\ v_k \end{bmatrix} + w_k$
  - Therefore $\boldsymbol{A}_k = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \boldsymbol{B}_k = \begin{bmatrix} 0 \\ \Delta t/m \end{bmatrix}, \boldsymbol{D}_k = \begin{bmatrix} 0 & 1 \end{bmatrix}$ (note $\boldsymbol{s}_k = \begin{bmatrix} r_k \\ s_k \end{bmatrix}$)
  - Therefore $\boldsymbol{Q}_k = \Delta t^2\mathbb{E}[\boldsymbol{s}_k\boldsymbol{s}_k^T]$; note the Δt^2, since the noise is scaled by Δt