Tutorial 2, Sep 20, 2023

Active Filters

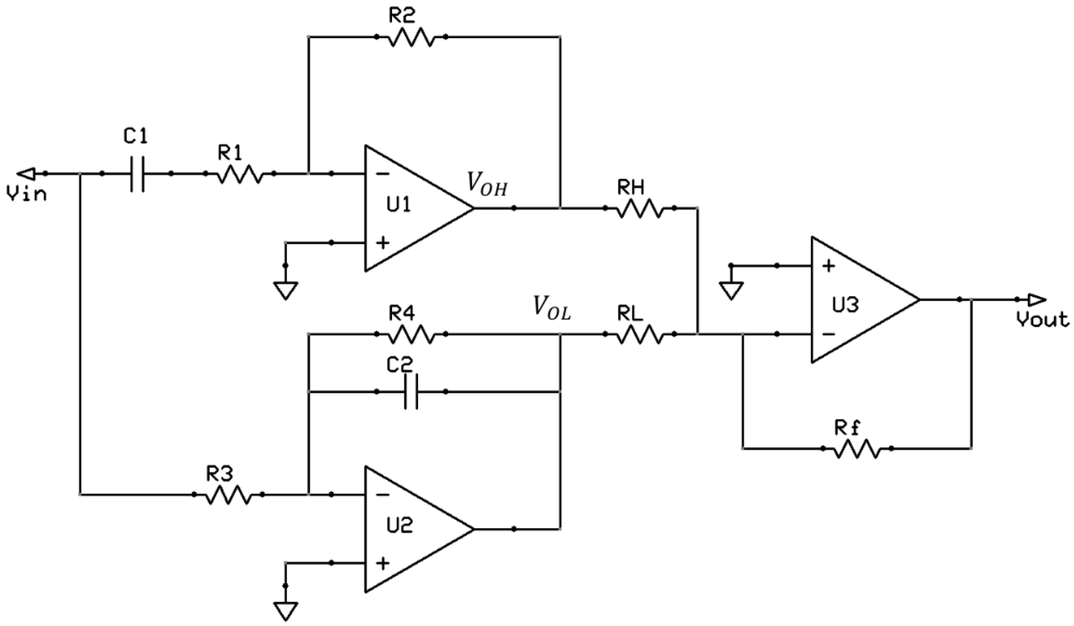

Figure 1: Example active bilinear circuit.

- Example: find the transfer function of the circuit above
  - Since this is an op-amp circuit, we can combine transfer functions by simply multiplying; we will first find the transfer functions for V_{OH} and V_{OL}
  - Recall that for an inverting amplifier configuration the transfer function is $H(s) = -\frac{Z_2}{Z_1}$ where Z_1 is the impedance before the input, and Z_2 is the feedback impedance
    * Both the op-amps at V_{OH} and V_{OL} are in an inverting amplifier configuration
    * For op-amp U_1 the input impedance is $Z_1 = \frac{1}{sC_1} + R_1$ and the feedback impedance is R_2, so $H_1(s) = -\frac{R_2}{\frac{1}{sC_1} + R_1} = -\frac{sR_2C_1}{1 + sR_1C_1}$
    * For op-amp U_2 the input impedance is $Z_1 = R_3$ and $Z_2 = \frac{1}{\frac{1}{R_4} + sC_2}$ so $H_2(s) = -\frac{1}{R_3(\frac{1}{R_4} + sC_2)} = -\frac{R_4}{R_3 + sR_3R_4C_2}$
  - The op-amp U_3 is in a *summing inverting amplifier* configuration
    * To find the transfer function for it, we can use superposition for each input separately
    * We find that the output is the sum of two inverting amplifiers, $V_{out} = -\frac{Z_F}{Z_1}V_{in_1} - \frac{Z_F}{Z_2}V_{in_2}$
    * Therefore $V_{out} = -\frac{R_f}{R_H}V_{OH} - \frac{R_f}{R_L}V_{OL}$
      $$= \left(\frac{R_f}{R_H}\frac{sR_2C_1}{1 + sR_1C_1} + \frac{R_f}{R_L}\frac{R_4}{R_3 + sR_3R_4C_2}\right)V_{in}$$
    * Overall this circuit essentially breaks the input signal into two parts, one higher in frequency and one lower, and sums them up and amplifies them; the proportion of higher frequency to lower frequency in the sum is controlled by the resistances R_H, R_L
- Example: find the transfer function of the circuit above

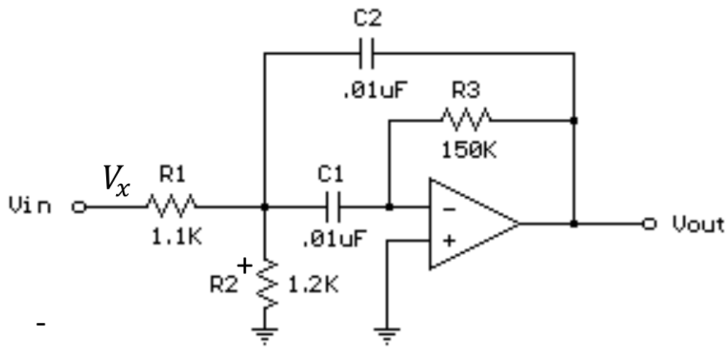

Figure 2: Example single op-amp bandpass filter.

- Let V_x be the voltage at the node to the left of the virtual ground
- $\dfrac{V_x - V_{in}}{R_1} + \dfrac{V_x}{R_2} + sC_2(V_x - V_{out}) + sC_1V_x = 0$
- At the virtual ground $-sC_1V_x - \dfrac{V_{out}}{R_3} = 0 \implies V_x = -\dfrac{V_{out}}{sR_3C_1}$
- We can then sub in V_x to find the transfer function through the equation at V_x
- $\dfrac{V_{out}}{V_{in}} = -\dfrac{sR_2R_3C_1}{R_1R_2 + R_2^2 + sR_1^2R_2C_2 + sR_1^2R_2C_1 + s^2R_1^2R_2R_3C_1C_2}$

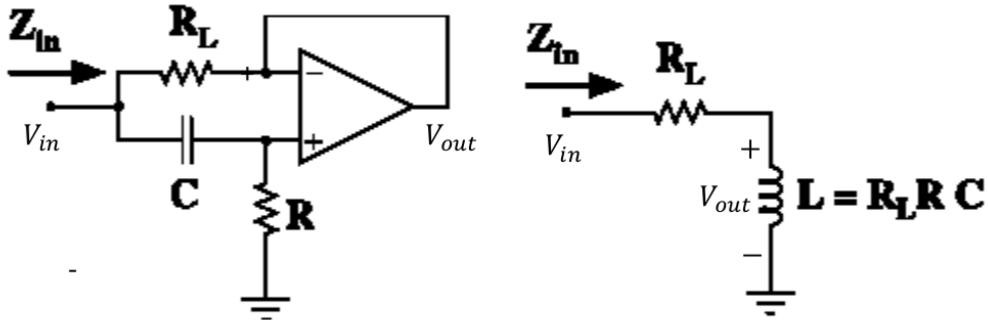

Figure 3: Example gyrator circuit.

- Example: find the transfer function of the circuit above and show that it is equivalent to the one on the right
  - At both inputs of the op-amp, the voltage is V_out
  - At the noninverting input: $sC(V_{out} - V_{in}) + \dfrac{V_{out}}{R} = 0 \implies V_{out}\left(sC + \dfrac{1}{R}\right) = V_{in}sC \implies \dfrac{V_{out}}{V_{in}} = \dfrac{sRC}{sRC + 1} = \dfrac{s}{s + \frac{1}{RC}}$
  - For the second circuit we have a voltage divider, so $V_{out} = \dfrac{L}{R_L + L}V_{in} = \dfrac{sL}{sL + RL} = \dfrac{sR_LRC}{sR_LRC + R_L} = \dfrac{sC}{sC + \frac{R_L}{R_LR_LR}}$