Tutorial 1, Sep 13, 2023

Transfer Functions

- Recall that the impedance for capacitors and inductors are dependent on frequency
  - Resistor: R
  - Capacitor: $\frac{1}{j\omega C} = \frac{1}{sC}$ where $s = j\omega$ and ω is the frequency
    - With increasing frequency, the impedance decreases
  - Inductor: $j\omega L = sL$
    - With increasing frequency, the impedance increases

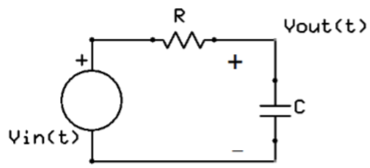

Figure 1: Example problem 1.

- Example problem 1: given the circuit above, what is $v_{out}(t)$ if $v_{in}(t) = u(t)$ (unit step)?
  - To do this we Laplace transform the input and use the transfer function
  - The impedance of the resistor is $Z_1 = R$ and the capacitor is $Z_2 = \frac{1}{sC}$; the two impedances form a voltage divider
  - $v_{out} = \frac{Z_2}{Z_1 + Z_2} v_{in} = \frac{1}{sC(R + \frac{1}{sC})} v_{in} = \frac{1}{sRC + 1} v_{in} \implies \frac{v_{out}}{v_{in}} = \frac{1}{sRC + 1} = H(s)$
  - The frequency domain signal is $\mathcal{L}\left\{u(t)\right\} = \frac{1}{s}$ so $Y(s) = \frac{1}{s} H(s) = \frac{1}{s(sRC + 1)} = \frac{1}{s} \cdot \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$
    - We can already see based on the poles that this would be a decaying exponential plus an offset
  - Partial fraction expansion: $\frac{A}{s} + \frac{B}{s + \frac{1}{RC}} = \frac{1}{s} \cdot \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \implies \left(s + \frac{1}{RC}\right) A + sB = \frac{1}{RC}$
    - Evaluate at $s = 0 \implies A = 1$
    - Evaluate at $s = -\frac{1}{RC} \implies B = -1$
    - $Y(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{RC}}$
  - Apply the inverse transform: $\mathcal{L}^{-1}\left\{Y(s)\right\} = y(t) = u(t) - u(t)e^{-\frac{t}{RC}}$

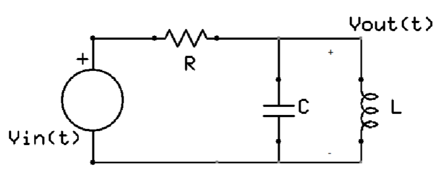

Figure 2: Example problem 2.

- Example problem 2: find the transfer function for the above circuit
  - Nodal analysis at v_{out}: $\frac{v_{out} - v_{in}}{R} + sCv_{out} + \frac{v_{out}}{sL} = 0 \implies \frac{v_{out}}{v_{in}} = \frac{s}{s^2 RC + s + \frac{R}{L}}$

Op-Amp Basics

- In practice, the op-amp requires both a positive and negative power input; however for ideal op-amps that we will analyze, they are not limited by power

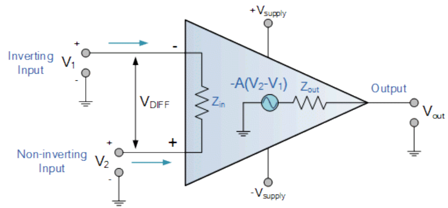

Figure 3: Diagram of an op-amp.

- It has two inputs, the noninverting input V_i^+ and inverting input V_i^-; the currents i^+, i^- going into the two inputs are both zero for an ideal op-amp
- The op-amp amplifies the voltage difference between the two inputs by the *gain* A: $V_o = A(V_i^+ - V_i^-)$
  - For an ideal op-amp, the gain A is effectively infinite, so op-amps are almost never used as-is; instead feedback loops are used
  - The output V_o would not be affected by any impedance (it maintains its voltage regardless of load)
  - In a real op-amp there is also an output impedance, but we will assume it is zero
- Typically, the inverting input is connected through a series of impedances to the output; this results in the voltage difference between the inputs being driven to zero
  - An input signal would be connected through an impedance into the inverting input, while the noninverting input is grounded or connected to a DC voltage; this configuration is called an *inverting amplifier*
  - When the noninverting input is grounded, the inverting input is forced to ground; we refer to it as a *virtual ground*

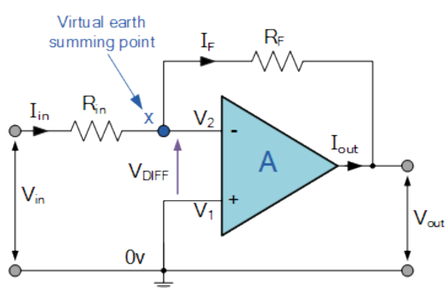

Figure 4: Inverting amplifier circuit.

- What is the transfer function of the amplifier circuit?
  - Notice that $i_{in} = i_F$ since there is no current going into the amplifier
  - $\dfrac{V_2 - v_{in}}{R_{in}} + \dfrac{V_2 - v_{out}}{R_F} = 0 \implies \dfrac{v_{in}}{R_{in}} - \dfrac{v_{out}}{R_F} = 0 \implies \dfrac{v_{out}}{v_{in}} = -\dfrac{R_F}{R_{in}}$
  - This circuit is called an inverting amplifier because of the negative sign; the input signal is amplified and inverted
  - Note that the value of v_{out} stays the same regardless of any load driven by it; if we have a two-stage amplifier, we can find the transfer functions for both stages separately and then multiply them together, since the second stage would not affect the output of the first stage