Lecture 9, Oct 6, 2023

From Bode Plot to Transfer Function

- We can reverse a Bode plot back into a transfer function by examining the magnitude plot:
  - Start by looking at the graph and finding all the inflection points – these are the pole and zero corner frequencies
  - Reverse the change in slope to find the multiplicities of the poles/zeroes
  - Check for poles and zeros at the origin
  - Pick any point on the graph, and use this to solve for the pure gain on the transfer function
  - Note if we started with an exact bode plot, we have to first estimate the asymptote plot (however this is relatively rare since reversing a bode plot is often used as a design tool)
- However, if we only look at the magnitude graph, we do not get a unique solution – there are multiple transfer functions that will give you the same magnitude plot, but not the same phase plot
  - Most of the time we don't care about the phase change when working with filter design
  - This could matter a lot in e.g. control systems
  - It also limits us to systems with magnitude rates of change being multiples of 20dB per decade; transfer functions with complex components can be problematic

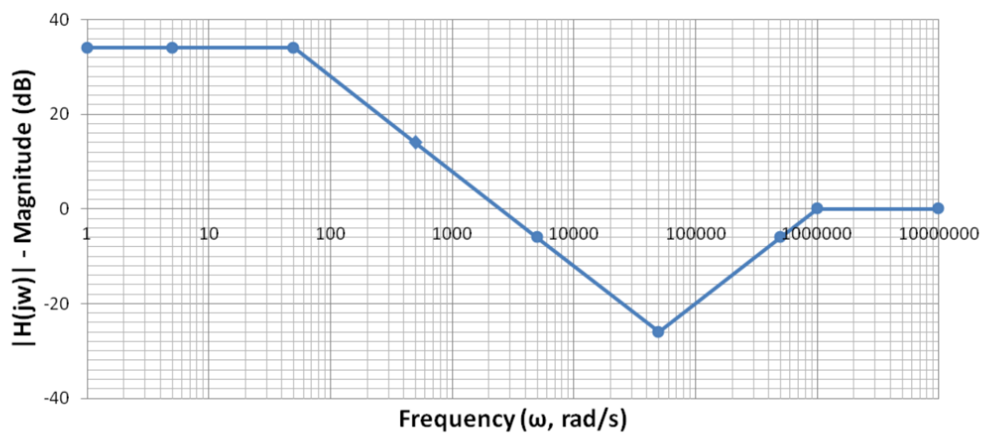

Figure 1: Phase plot for the example problem.

- Example reverse the Bode phase plot above:
  - We see inflection points at 50, 50k, and 1M, which are the corner frequencies
  - At 1M, the slope changes from +20dB per decade to 0, so a pole with multiplicity 1 became active
  - At 50k, the slope changes from −20dB per decade to +20dB per decade, so a zero with multiplicity 2 became active
  - At 50, the slope changes from 0 to −20dB per decade, so a pole with multiplicity 1 became active
  - We started with a flat slope, so there is no pole or zero at the origin
  - Therefore the transfer function has form $\dfrac{|K|(s+50000)^2}{(s+50)(s+1000000)}$
  - At $\omega = 1$ the magnitude plot has value 34dB = 50.119; in our transfer function we have $|K|(50000)^2\left(\dfrac{1}{50}\right)\left(\dfrac{1}{1000000}\right) = 50K$
    * Note we could choose this because the first corner frequency is more than a decade greater
  - Therefore $|K|$ is about 1, so $H(s) = \dfrac{(s+50000)^2}{(s+50)(s+1000000)}$
    * Note that we could've set $K = \pm 1$ and we would get the same magnitude; this would be reflected in the phase plot

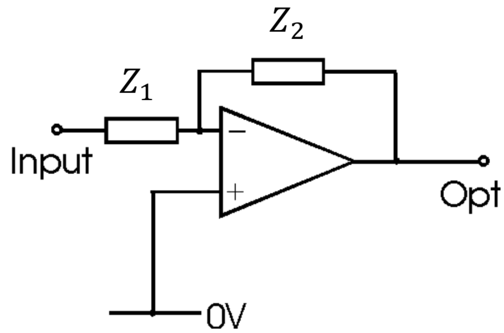

Figure 2: The inverting amplifier as a building block.

From Transfer Function to Circuit

- Due to our simplifications, the simplest way for us to do this is to use a bunch of op-amp building blocks
  - Note this is an easy way, but not an optimal way
- Each inverting amplifier can (usually) give us up to one pole and one zero
- To get the desired Z_1, Z_2 to make the poles, we can choose a resistor and either an inductor or a capacitor
  - In practice capacitors are preferred because they are much cheaper and available in a wider range of values
  - However capacitors have a shorter lifespan (tens of thousands of hours vs. decades for inductors)
  - Capacitors are better at the lower frequencies while inductors are better at higher frequencies
  - In addition, we can also do resistors and capacitors/inductors in series or parallel, giving us a total of 4 choices per impedance
    * This means 16 possible combinations! Some of these can implement more advanced functions such as double poles/zeros
  - Example combination:
    * Both resistor and capacitor in series: $Z_1 = R_1 + \frac{1}{sC_1}, Z_2 = R_2 + \frac{1}{sC_2}$, then the transfer function has a pure gain of $K = -\frac{R_2}{R_1}$, a pole at $\frac{1}{R_1C_1}$, and a zero at $\frac{1}{R_2C_2}$
    * Both resistor and inductor in series: $Z_1 = R_1 + sL_1, Z_2 = R_2 + sL_2$, then the transfer function has a pure gain of $K = -\frac{L_2}{L_1}$, a pole at $\frac{R_1}{L_1}$ and a zero at $\frac{R_2}{L_2}$
- In the real world, we need to choose realistic component values:
  - Resistors from 100Ω to 4.7MΩ
    * Too low and we'll get opamp loading effects
    * Too high and there will be current going into the opamp
  - Capacitors from 10pF to 1μF
    * Too low and it will be too hard to make and too sensitive (capacitance exists between rows on a breadboard!)
    * Too high and we'll have to use electrolytic capacitors, which are polarized, and less accurate
  - Inductors from 1μH to 500mH
    * Too low and the inductance will be comparable to PCB traces, so the circuit will be extremely sensitive
    * Too high and the inductor will be too hard to make and too big
- Systematic procedure to find a circuit:
  1. Group poles and zeros into pairs; each pair uses an inverting amplifier block

    - Try to keep the corner frequencies of the poles and zero close
    - When poles and zeros are very different, the gain will be extreme and reduces flexibility
    - For any remaining lone poles and zeros, add another amplifier block
2. Divide any pure gain among the blocks; add additional pure gain blocks as needed
    - We can estimate the amount of gain that a stage provides by dividing the zero by the pole, so we can get an estimate of how much gain is left
    - Remember that real opamps have gain limits
3. Select realistic component values
    - Start with capacitors and inductors first because they have a much smaller range of values

- Example: $H(s) = \frac{(s+50000)^2}{(s+50)(s+1000000)}$
    - We have to match one 50k zero with the 50 pole and the other 50k zero with the 1M pole
    - The first stage has a gain of approximately 1000, the second has a gain of 1/100, which leaves us with a gain of 10
        * When the leftover gain is one or two magnitudes, we are usually able to divide it among all the stages without having to add an additional amplifier
        * Note in practice we need to keep track of the magnitude of our signal in-between stages; if the signal becomes too small, it can get lost among the noise; if it's too big, it can get clipped
            - We might want to shuffle around the stages; e.g. if we have 2 stages with really big gain and 2 stages with really small gain, we should alternate the big and small gains so the signal does not get lost or clipped
    - If we have more freedom in grouping poles and zeros, we can try to group them differently in order to reduce the leftover gain
    - Lower frequency poles and zeroes are more easily realized with capacitors; higher frequency poles and zeros are more easily realized with inductors (this is a direct result of the range of component values we can use)
    - In the real world we might want to calculate the equivalent impedance of each stage of the circuit to prevent loading effects
    - For now we will try using only resistors and capacitors, by RC series
    - $H_1(s) = K_1\frac{(s+50000)}{(s+50)}, H_2(s) = K_2\frac{(s+50000)}{(s+1000000)}$ so $K_1K_2 = 1$ which is our constraint
        * We can try to get K_1, K_2 as close to 1 as possible for both stages
    - For circuit 1: we can try to select C_1, C_2 first
        * We want $\omega_c = 50\text{rad/s}$ which has value $P = \frac{1}{R_1C_1}$
        * We can try a capacitor value that's in the middle of the range, e.g. 0.5μF, giving $R_1 = 43\text{k}\Omega$
    - For circuit 2: let's pick $R_2 = R_1$
        * We get a capacitor value around 23pF, which works but is quite small

Important

Lower frequency poles and zeroes are more easily realized with capacitors; higher frequency poles and zeros are more easily realized with inductors. This is a direct result of the range of component values we can use. Inductors begin to struggle below 100.