Lecture 5, Sep 22, 2023

Analyzing Complex Op-Amp Circuits

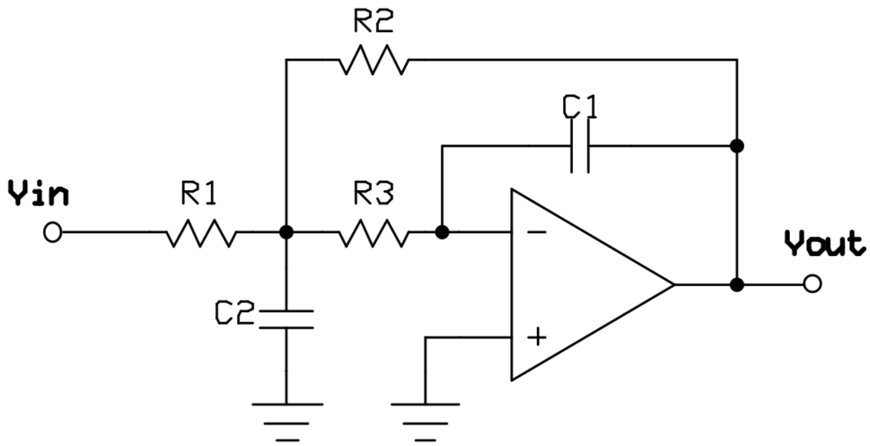

Figure 1: An inseparable op-amp circuit.

- Example: find the transfer function of the above circuit
  - There is only one op-amp so we can't break this circuit down; we must write multiple node equations
  - We should write node equations at the inverting input (V_y) and the node to the right of V_{in} (V_x)
    * We also know that $V_y = 0$
  - $\dfrac{V_x - V_{in}}{R_1} + sC_2V_x + \dfrac{V_x - V_{out}}{R_2} + \dfrac{V_x}{R_3} = 0 \implies \left(sC_2 + \dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{1}{R_2}\right)V_x + \left(-\dfrac{1}{R_1}\right)V_{in} + \left(-\dfrac{1}{R_2}\right)V_{out} = 0$
  - $\dfrac{-V_x}{R_3} + -sC_1V_{out} = 0 \implies V_x = -sR_1C_3V_{out}$
  - Sub back to get $H(s) = \dfrac{\left(\frac{1}{R_1}\right)}{-s^2C_1C_2R_3 + \left(-\frac{C_1R_3}{R_1} - C_1 - \frac{C_1R_3}{R_2}\right)s + \left(-\frac{1}{R_2}\right)}$
    * Normalize: $-\dfrac{\frac{1}{C_1C_2R_1R_3}}{s^2 + \left(-\frac{1}{C_2R_1} + \frac{1}{C_2R_2} + \frac{1}{C_2R_3}\right)s + \left(\frac{1}{C_1C_2R_2R_3}\right)}$

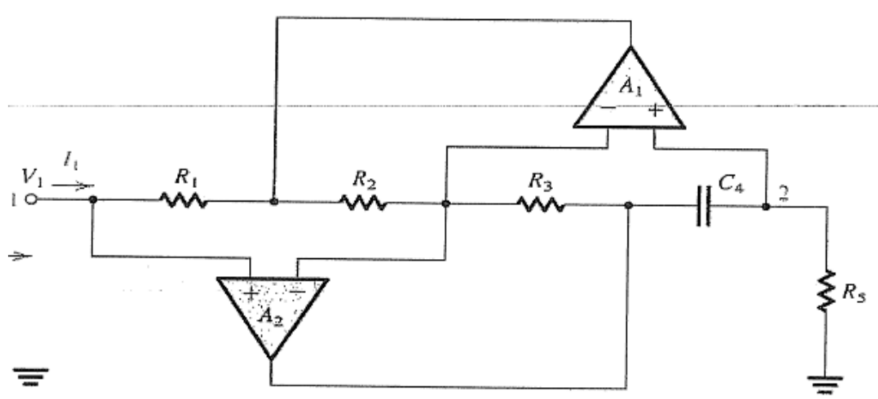

Figure 2: Example circuit to find the equivalent impedance.

- Example: find the equivalent input impedance of the circuit above
  - We want to find an input current I_{in} caused by an input voltage V_{in}, then we can find the impedance as $\frac{V_{in}}{I_{in}}$
  - Note we can't write useful node equations at the node between R_1, R_2 or the node between R_3, C_4 because they are both connected to the output of op-amps
  - The node between R_2, R_3 and the node between C_4, R_5 have the same voltage as V_{in} due to the op-amp feedback
  - Node equations:
    * $\frac{V_{in} - V_A}{R_1} - I_{in} = 0 \implies V_A = V_{in} - R_1 I_{in}$
    * $\frac{V_{in} - V_A}{R_2} + \frac{V_{in} - V_C}{R_3} = 0 \implies V_{in}\left(\frac{1}{R_2} + \frac{1}{R_3}\right) + V_A\left(-\frac{1}{R_2}\right) + V_C\left(-\frac{1}{R_3}\right) = 0 \implies V_C = V_{in} - \frac{R_1 R_3}{R_2} I_{in}$
    * $sC_4(V_{in} - V_C) + \frac{V_{in}}{R_3} = 0 \implies -\frac{sC_4 R_1 R_3}{R_2} I_{in} + \frac{1}{R_5} V_{in} = 0$
    * $Z_{eq} = \frac{V_{in}}{I_{in}} = s\left(\frac{C_4 R_1 R_3 R_5}{R_2}\right)$
  - If we let $\frac{C_4 R_1 R_3 R_5}{R_2} = L$, we see $Z_{eq} = sL$ – this circuit is an inductance simulator
  - Inductors are very hard to work with at a small scale, so circuits like these are often used to replace inductors with other components
  - Note this only simulates the frequency response of an inductor, but not other aspects like the energy storage

Real-World Op-Amps

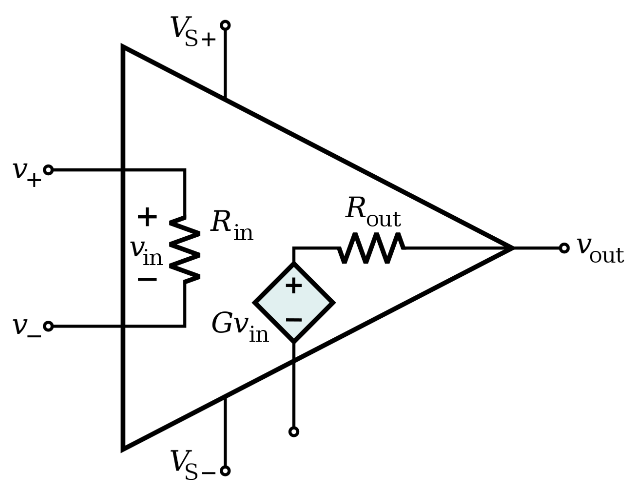

Figure 3: Equivalent internal model of a real-life op-amp.

- Real-world op-amps have several differences compared to ideal op-amps:
  1. Feedback loops: the degree of negative feedback (output to inverting input) must be greater than the degree of positive feedback (output to noninverting input) for the system to be stable
     - For our purposes, we assume that the feedback is sufficient as long as there is a path from the output to the inverting input
  2. Finite input impedance: the impedance looking into the terminals is finite, which means that a small current can flow into the inputs
     - This impedance is frequency dependent but most of the time it suffices to model as a simple resistance

    - The input impedance is often in the range of megaohms to gigaohms, even for cheap op-amps
    - In most cases, the assumption that no current flows in is valid, unless the external resistances attached are on the same order of magnitude (or greater) than the internal resistance
3. Nonzero input offset: the difference between the voltages on the terminals is nonzero due to the finite open-loop gain
    - This is often in the range of millivolts or nanovolts, so it will not affect the circuit in most cases
    - This does not matter most of the time, but for particular sensitive circuits or filters it needs to be accounted for
4. Finite open-loop gain: the op-amp output voltage is $v_{out} = (v_+ - v_-)A_0$ where A_0 is the open-loop gain; in a real-world op-amp, this gain is finite
    - This means that the feedback control is not perfect, and so the difference between the voltages of the two inputs is nonzero
    - In modern op-amps, the gain is often around 100,000 for cheaper modern op-amps, or up to millions for better ones
        * For older technology this could be in the thousands
    - To see if this will have an impact, we need to compare the op-amp gain to the function of the circuit we're trying to implement
        * e.g. if we're trying to make an inverting amplifier, and the gain we want is within an order of magnitude of the open-loop gain, then it could be problematic
5. Finite bandwidth, conditional stability, and not linear time-invariant (LTI): real op-amps can be frequency-limited, can have stability issues, and may not be perfectly linear time-invariant
    - The bandwidth is usually limited to megahertz to gigahertz ranges
    - The op-amp should be at least an order of magnitude faster than the operating range of the circuit
6. Nonzero output impedance: the impedance of the output is nonzero, so the amount of current we can draw is finite
    - Drawing too much current would start causing considerable loading effects
    - The output resistance is typically in the range of ohms to kilohms
    - Since we're usually using kilohm-level resistances in our circuits, this could be a very common issue

- Additionally, real-world op-amps must be powered by some pair of voltages V_{dd}, V_{ss}, which restricts the range that V_{out} can take on
    - If we expect the output voltage to be outside this range, it is capped instead
    - When the waveform is cut off due to op-amp output limits, it is referred to as *clipping*
    - Clipping may not happen right at the supply voltage; there may be a small gap and clipping happens slightly below the supply voltage