Lecture 4, Sep 20, 2023

Transfer Functions of Op-Amp Circuits

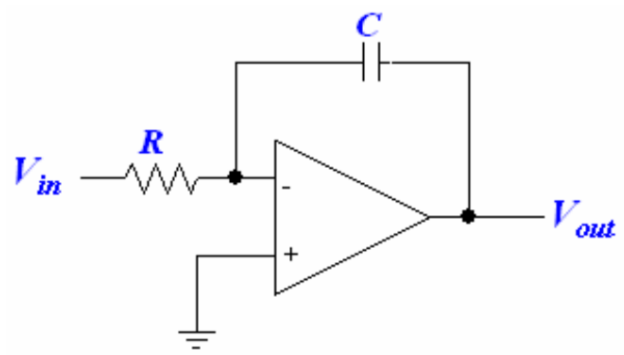

Figure 1: Example integrator op-amp circuit.

- Example: find the transfer function for the circuit above
  - As a shortcut, we can notice how this resembles an inverting amplifier, so we can use the same formula and replace the resistances with impedances
    * Recall that for an inverting amplifier $\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$, so in this case we have $H(s) = -\frac{Z_2}{Z_1} = -\frac{1}{sRC}$
  - If we couldn't rely on the shortcut, we must use KVL at the inverting input
    * $\frac{V_x - V_{in}}{R} + \frac{V_x - V_{out}}{\frac{1}{sC}} = 0$
    * Since there is a valid negative feedback path, we know the voltage at the inverting and noninverting inputs are equal; therefore $V_x = 0$
    * Using this, $-\frac{V_{in}}{R} - sCV_{out} = 0 \implies \frac{V_{out}}{V_{in}} = -\frac{1}{sRC}$
  - We couldn't have used KVL at the output node, because the op-amp has a current output that we do not know
  - Since multiplying a signal by $\frac{1}{s}$ in the frequency domain is equivalent to an integration (because multiplication by s is differentiation), this circuit is an integrator for the input signal
  - Note: in practice, capacitors don't hold their charge perfectly; since the capacitor is effectively keeping track of the integration state, the capacitor leaking charge leads to errors; in addition, real noise is often not zero-mean, so noise in the signal may accumulate

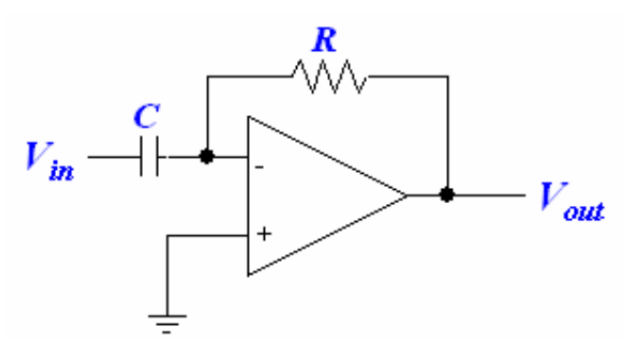

Figure 2: Example differentiator op-amp circuit.

- Example: find the transfer function for the circuit above

- $H(s) = -\frac{Z_2}{Z_1} = -\frac{R}{\frac{1}{sC}} = -sRC$
- Multiplication by s is a differentiation in time domain, so this circuit is a differentiator
- Note that this and the integrator are frequency limited; they may not respond properly with very high frequencies

- When there are multiple inputs, we can find the transfer functions for each input separately; however we have to assume that the other inputs are at some fixed known voltage, usually zero

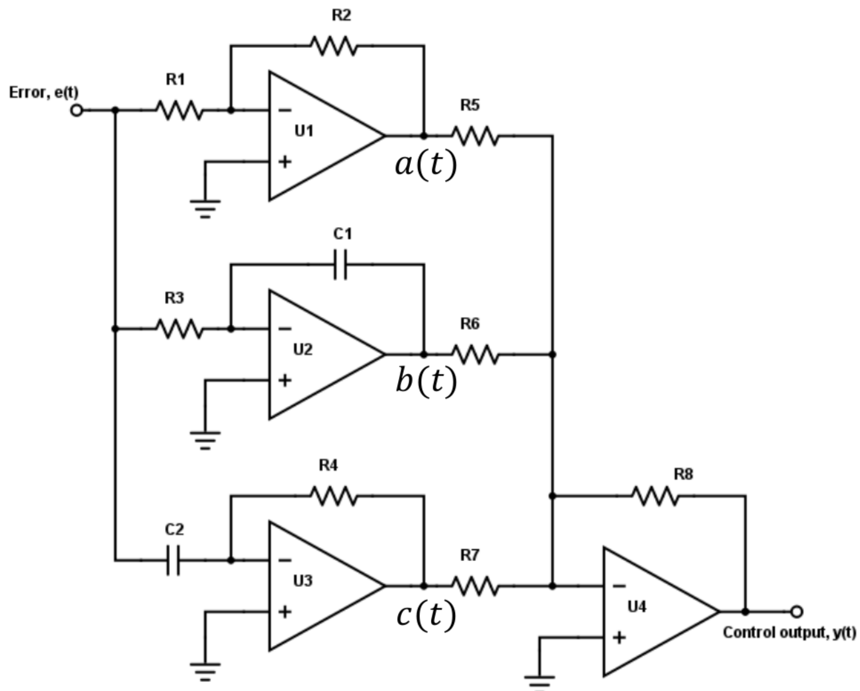

Figure 3: Example PID control loop circuit.

- Example: find the transfer function for the circuit above
  - Notice that we can break this into the 4 individual op-amp circuits: the inverting amplifier U_1, the integrator U_2, the differentiator U_3, and summing amplifier U_4; we will find transfer functions for each one individually and combine them
  - Note: this is a PID circuit, where an input error is scaled, integrated, and differentiated, and then combined together to form the control output
  - $H_1(s) = -\frac{R_2}{R_1}, H_2(s) = -\frac{1}{sR_3C_1}, H_3(s) = -sR_4C_2$
    * Note our assumptions: no loading effects at the input or output
  - For the summing amplifier, $Y(s) = -\left(\frac{R_8}{R_5}A(s) + \frac{R_8}{R_6}B(s) + \frac{R_8}{R_5}C(s)\right)$
  - Therefore $H(s) = \frac{Y(s)}{E(s)} = \frac{R_2R_8}{R_1R_5} + \frac{R_8}{R_3R_6C_1}\frac{1}{s} + \frac{R_4R_8C_2}{R_7}s$
  - Bringing this back to the time domain, we have $y(t) = K_Pe(t) + K_I\int e(t)\,\mathrm{d}t + K_D\frac{\mathrm{d}}{\mathrm{d}t}e(t)$