Lecture 20, Nov 24, 2023

Example: Common Source Amplifier

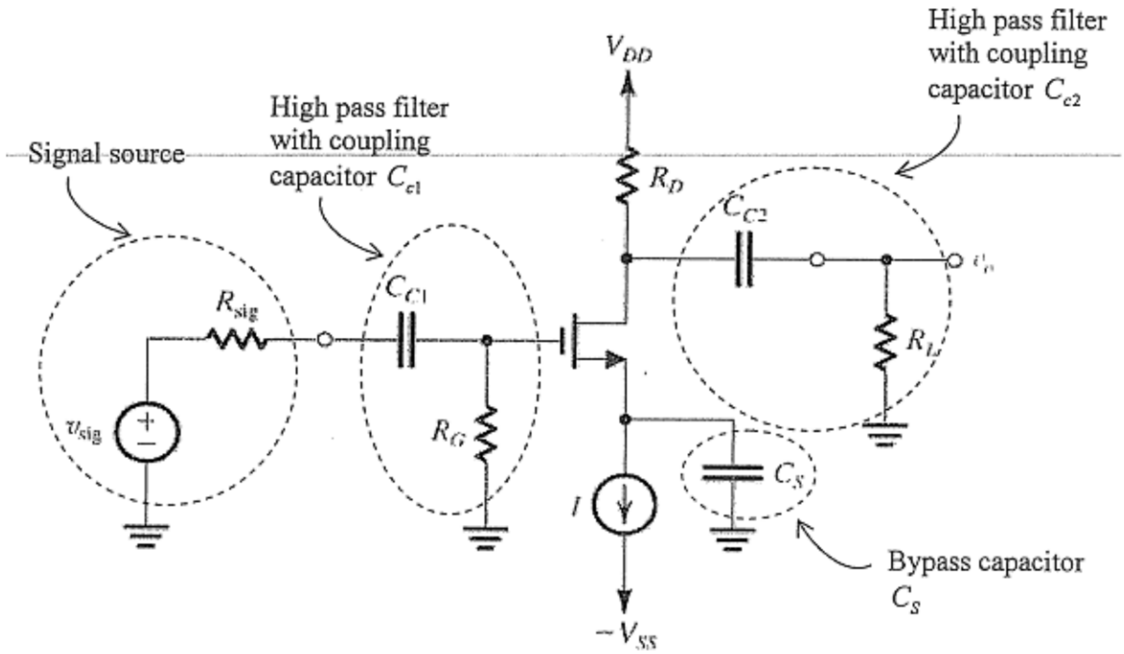

Figure 1: Example: Common source amplifier.

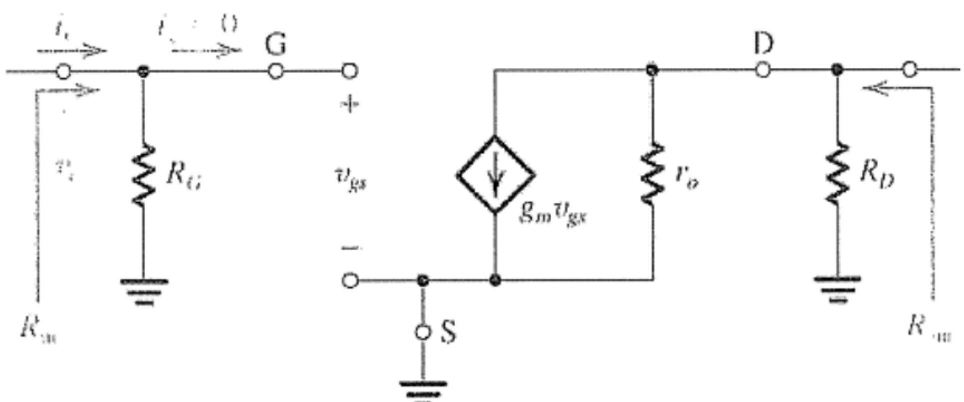

Figure 2: Transformed circuit of the example.

- Example: find the open circuit voltage gain, input resistance, and output resistance of the amplifier above
  - First transform the circuit:
    * Replace coupling capacitors$ C_{C1}, C_{C2} and bypass capacitor C_S with short circuits
    * Everything in the first circle is the driving input source; this is what gives us the time-varying signal, which gets driven through some finite output impedance
      - This is not a part of our amplifier, so quantities here should not appear in our solution
      - We define v_{in} at the node right after this, between R_{sig} and C_{C1}
    * The circle at the top right is the load, so like the input source, we also don't consider it as a part of our amplifier

* Short circuit the voltage source at V_{DD} and replace the current source with an open circuit
  * Notice that if we didn't have the bypass capacitor, the circuit would not work, because after removing the current source the source would be floating
* Replace the MOSFET with the default model

– We will start by calculating the open-circuit voltage gain; for this we assume no loading effects whatsoever
  * Later on we will account for loading effects externally, so for now we won't consider it
  * Start by writing a node equation at v_{out} (the drain): $\frac{v_{out}}{R_D} + \frac{v_{out}}{r_0} + g_m v_{gs} = 0$
    * Note since we're calculating open loop gain, there's no current flowing in or out of the output
  * $v_{gs} = v_g - v_s = v_{in} - 0 = v_{in}$ so we have $\frac{v_{out}}{R_D} + \frac{v_{out}}{r_0} + g_m v_{in} = 0$ which we can now use to solve for $A_{v_0} = \frac{v_{out}}{v_{in}}$
  * This works out to $A_{v_0} = -g_m(r_0 \parallel R_D)$ or $-g_m R_D$ if ignoring channel length modulation
  * Note that since g_m and R_D are always positive, this is an inverting amplifier

– Generally, common source amplifiers have high open-loop gain (more amplification) and high input resistance (loads input less), but generally high output resistance (susceptible to output loading)
  * These are often used near the input – the large gain bumps the signal up from the noise floor and the high input resistance is good

– Now find the input resistance $R_{in} = \frac{v_{in}}{i_{in}}$
  * For this circuit, it is trivial: $R_{in} = R_G$ since no current goes through the gate
  * We can simply take a large R_G to increase the input resistance
    * R_G is part of the DC bias point but not any of the other variables, so we can more or less take it to be anything we want

– The output resistance is $R_{out} = \frac{v_{out}}{i_{out}}$
  * Note we need to assume some value for v_{in} to do this calculation
  * We assume $v_{in} = 0$ to avoid double counting gain effects (i.e. we assume overall input voltage is constant)
  * If $v_{in} = 0, v_{gs} = 0$ so we get no current through the current source
  * Therefore the equivalent resistance is $r_0 \parallel R_D$ or R_D without CLM

– Notice that we need a high R_D to get high open loop gain, but this raises the output resistance

Amplifier Equivalent Models

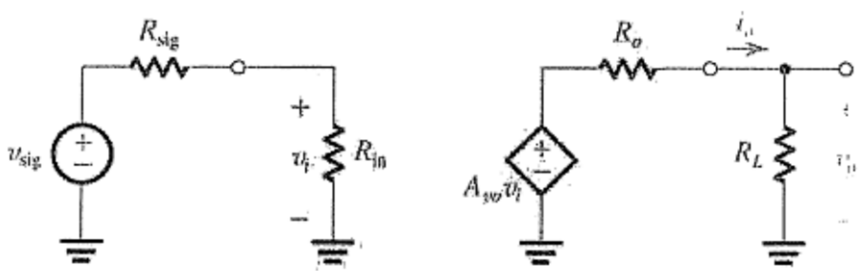

Figure 3: Equivalent model for the amplifier we solved above.

* The amplifier circuit we solved for above could be replaced by the equivalent model above, where the amplifier is in the middle; using this we can completely abstract away the amplifier
* The amplifier is replaced by R_{in} to ground, the voltage source $A_{v_0}v_{in}$ and output resistance R_{out}
  – Notice that R_{in} forms a voltage divider with R_{sig} and R_{out} with R_L
  – Both loading effects will reduce A_{v_0}; we only get this total gain with a zero R_{sig} and infinite R_L

- Otherwise the actual gain is $A_{v_{\text{total}}} = \left(\frac{R_{in}}{R_{in}+R_{sig}}\right) A_{v0} \left(\frac{R_L}{R_L+R_{out}}\right)$

Example Amplifiers

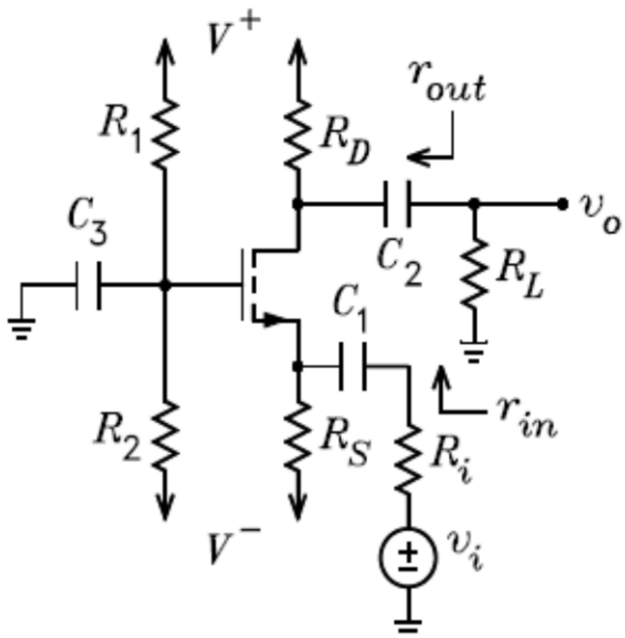

Figure 4: Example: Common gate amplifier.

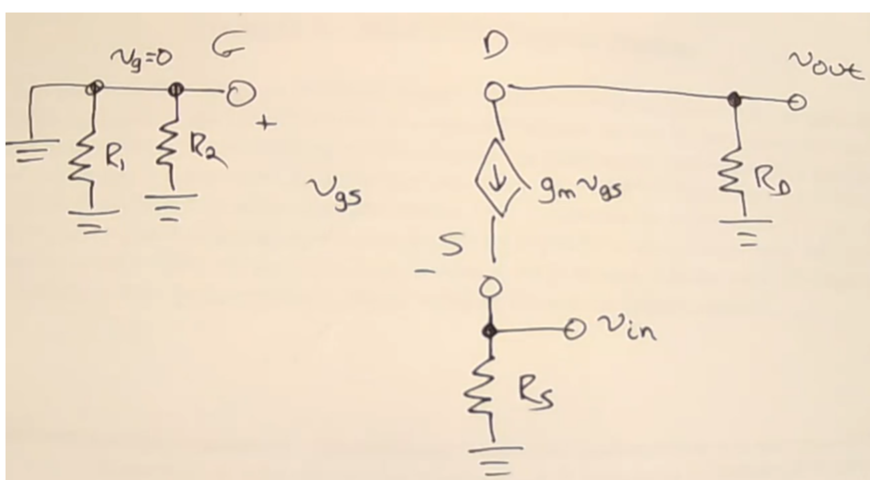

Figure 5: Transformed circuit.

- Example: Find the same 3 quantities in the circuit above, ignoring channel length modulation
  - First transform the circuit
    * R_L is part of the load and R_i is part of the source, so both can be removed
    * Replace C_1, C_2, C_3 with short circuits, V^+ and V^- to ground
    * Replace the MOSFET with the default model
  - Note that after shorting the capacitors, $v_g = 0$ so we can effectively ignore R_1, R_2
  - For A_{v_0} we can write a node equation at v_{out}: $\frac{v_{out}}{R_D} + g_m v_{gs} = 0$

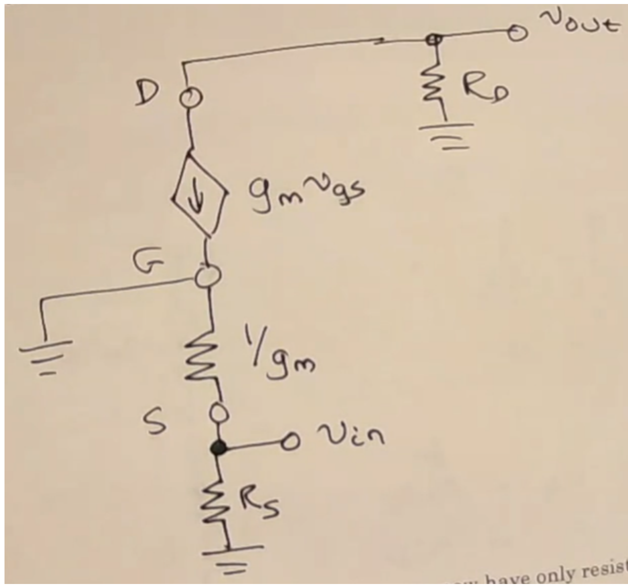

Figure 6: Transformed circuit using the T-model.

    - $v_{gs} = v_g - v_s = 0 - v_{in}$
    - Therefore $\frac{v_{out}}{R_D} - g_m v_{in} = 0$
    - $A_{v_0} = g_m R_D$
  - Notice that the open-loop gain is positive, so this is a non-inverting amplifier
  - Furthermore we can tweak g_m and R_D to potentially get a large open-loop gain
  - R_{in} and R_{out} are more complicated to solve due to the current source; we can redraw the circuit with the T-model to simplify things
  - With the T-model, we can easily tell that $R_{in} = R_S \parallel \frac{1}{gm}$
  - For R_{out}, with a $v_{in} = 0$ we have no current through the current source, so $R_{out} = R_D$

- Example: similar to the common source amplifier we solved before, but using a current mirror and no bypass capacitor
  - Note that this is not a proper amplifier, since we do not have a bypass capacitor
  - Normally this would not be solvable if we replaced the current mirror with a current source, but real world CLM effects provide a path for current
  - Use the same procedure to transform this circuit:
    - Replace V_{DD}, V_{SS} with shorts to ground
    - Remove input source and load
  - Notice that v_{gs_1} is a function of v_{in}, but v_{gs_2} and v_{gs_3} are not
    - The sources of both of these MOSFETs are grounded, and there is no path from v_{in} to their gates
    - In this scenario, we can assume $v_{gs_2} = v_{gs_3} = 0$ to simplify the circuit
  - Now we can eliminate the current source from the second MOSFET, leaving us with only r_{0_2}, a large resistance to ground
  - This is equivalent to having a source resistor on the original MOSFET circuit
    - Source resistors usually act like localized negative feedback – they reduce gain but improve some other parameters, such as lowering output resistance
    - However, unlike a normal source resistor, the CLM resistor is both hard to control and way too big
  - Write node equations:
    - At v_{s_1} or v_{D_2}: $\frac{v_{s_1}}{r_{0_2}} + \frac{v_{s_1} - v_{out}}{r_{0_1}} - g_{m_1} v_{gs_1} = 0$

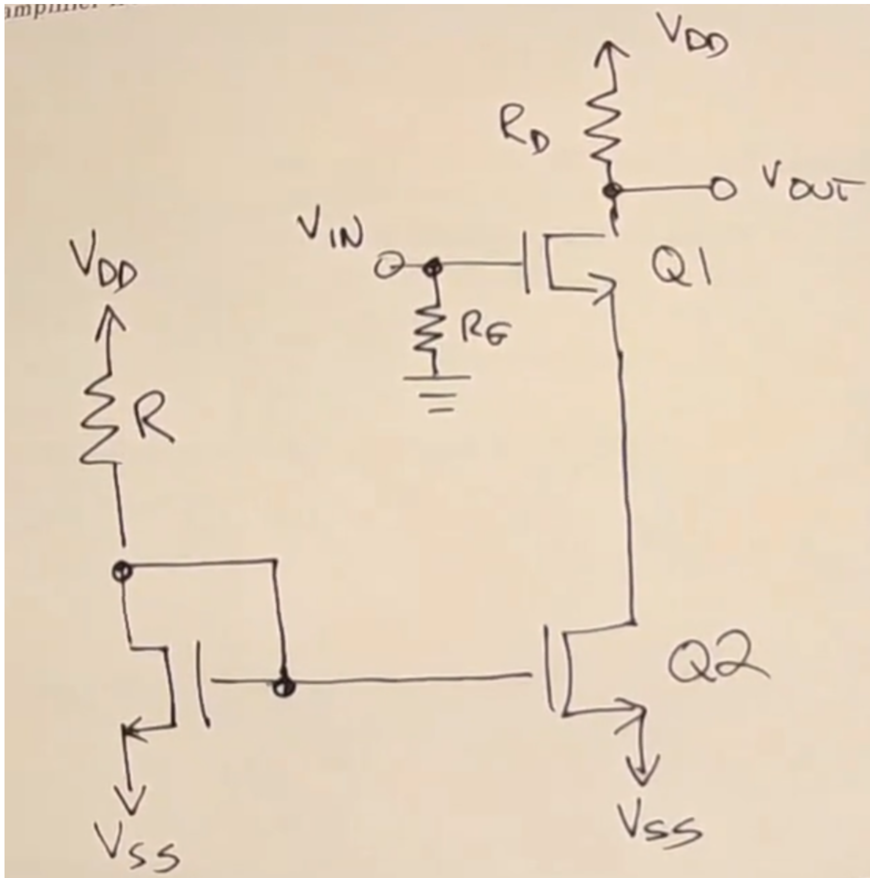

Figure 7: Second example circuit.

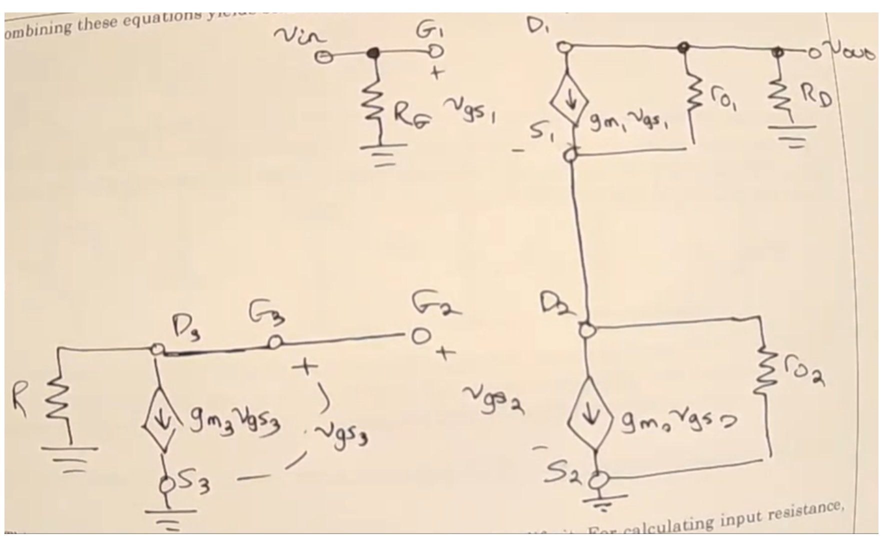

Figure 8: Transformed second example circuit.

- Note $v_{gs_1} = v_{in} - v_{s_1}$

* At v_{out}: $\frac{v_{out}}{r_{0_1}} + \frac{v_{out} - v_{s_1}}{r_{0_1}} + g_{m_1} v_{gs_1} = 0$
* Add the equations: $\frac{v_{s_1}}{r_{0_2}} + \frac{v_{out}}{r_{0_1}} = 0 \implies v_{s_1} = -\frac{r_{0_2}}{R_D} v_{out}$
* Substitute into one of the equations, and $A_{v_0} = \frac{v_{out}}{v_{in}} = -\frac{g_{m_1} r_{0_1} R_D}{g_{m_1} r_{0_1} r_{0_2} + r_{0_1} + r_{0_2} + R_D}$

– Sine the CLM resistances are high, this is approximately $\frac{R_D}{r_{0_2}}$
  * As expected, the negative feedback reduces the gain, approximately by dividing by the resistance value
  * However since r_{0_2} is much bigger than R_D, we will have a gain much less than 1, so this is not practical
  * This is why source resistors are typically in the ohms or hundreds of ohms range

– Input resistance is simply R_G, which has not changed

– Recall that when calculating output resistance only, we assume an input voltage of zero to avoid double-counting gain effects
  * Assuming $v_{in} = 0$ gives us $v_{gs_1} = -v_{s_1}$, which is not necessarily zero
  * We can brute force this by using a test output current
  * However in this case we have a *cascode connection*, we can replace the entire thing with a single resistor to ground R_A
  * $R_A = (1 + g_{m_1} r_{0_1}) r_{0_2} + r_{0_1} \approx g_{m_1} r_{0_1} r_{0_2}$
  * Therefore the output resistance is simply $R_A \parallel R_D$, but since R_D is much smaller than R_A, the output resistance is still about R_D

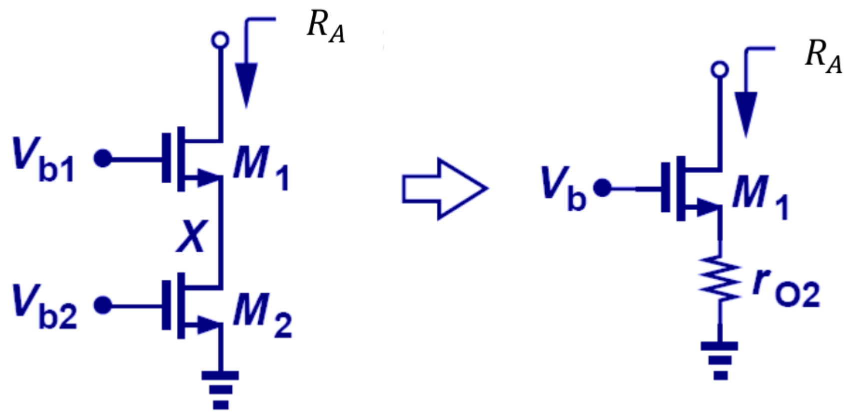

Figure 9: Cascode connection.