# Lecture 10, Oct 11, 2023

## Circuit From Transfer Function Example

- $H(s) = -\dfrac{s(s+100)(s+10000)}{(s+10)(s+500)(s+1000)}$
- Note the features:
  - Zero at the origin
  - Negative pure gain with $|K| = 1$, so $|K_1||K_2||K_3| = 1$, therefore we need an odd number of inverting op-amp stages
    * Note if we only care about matching the magnitude plot, then the negative sign doesn't matter for us
- Group the poles and zeros into the 3 stages:
  - Note we want to group them so that the distance between the pole and zero is minimized in each pairing
  - $H_1(s) = K_1\dfrac{s}{s+10}, H_2(s) = K_2\dfrac{s+100}{s+500}, H_3(s) = K_3\dfrac{s+10000}{s+1000}$
  - Splitting the pure gains among the stages, we want to keep the gain of each stage close to 1
- First stage: $H_1(s) = K_1\dfrac{s}{s+10}$
  - Note if we try an RC series combination, we will have a zero at $\dfrac{1}{R_1C_1}$; to achieve a zero at the origin, we need an infinite $R_1$, which means the signal cannot get through
    * Note we could still use RC parallel
  - Try an RL series combination: $H(s) = -\dfrac{L_2}{L_1}\dfrac{s+\frac{R_2}{L_2}}{s+\frac{R_1}{L_1}}$
    * $\dfrac{R_2}{L_2} = 0 \implies R_2 = 0$, i.e. $R_2$ will be a short; $L_2$ can be chosen freely, so we can match it with $L_1$ to achieve a gain of 1
    * $\dfrac{R_1}{L_1} = 10$ but this is unachievable!
      - Note if we try our largest possible inductor of 500mH, it would still require a resistor as small as 5Ω, which is outside our range
      - The small resistor means a very low input impedance in this stage, which could lead to considerable loading effects on whatever input goes into this stage
    * Note we could have anticipated this since 10 is a relatively small pole location, so a capacitor would do better (whereas an inductor would do better for a higher frequency pole)
    * To fix this, we could:
      - Try a more complex combination of RLC (i.e. mixing up inductors and capacitors in the feedback/input path); this is the most optimal but complex
      - Break this into two stages; this is a valid on a test but is suboptimal
      - Change the matching of poles and zeroes (swapping the pole at 10 with a higher frequency one)
    * e.g. we can swap the pole at 10 with a pole at 500
      - $H(s) = \dfrac{s}{s+500}$
      - $\dfrac{R_1}{L_1} = 500 \implies L_1 = 200\text{mH}, R_1 = 100\Omega$, which is marginally acceptable
      - Choose $L_2 = L_1 = 200\text{mH}$ so this stage has a pure gain of -1 (note we can come back and pick another value later to balance out the gains)
      - In reality, we should probably use the pole at 1000 instead so we can get less extreme values
- Second stage: $H_2(s) = K_2\dfrac{s+100}{s+10}$ (note we swapped in the pole from the previous stage)
  - The low frequency pole is difficult to realize with any RL combination, so we will use an RC combination

- Using RC series: $H_2(s) = \frac{R_4}{R_3} \frac{s + \frac{1}{R_3C_3}}{s + \frac{1}{R_4C_4}}$
    - $\frac{1}{R_3C_3} = 100 \implies C_3 = 0.1\mu\text{F}, R_3 = 100\text{k}\Omega$
    - Try setting $R_4 = R_3 \implies \frac{1}{R_4C_4} = 10 \implies C_4 = 1\mu\text{F}$
- Third stage: $H_3(s) = K_3 \frac{s + 10000}{s + 1000}$
  - In this case both the zero and pole are near the range where capacitors and inductors work well
  - Use RC series again: $H_3(s) = \frac{R_6}{R_5} \frac{s + \frac{1}{R_5C_5}}{s + \frac{1}{R_6C_6}}$
    - $C_6 = 1\text{nF}, R_6 = 100\text{k}\Omega$
    - $R_5 = R_6 = 100\text{k}\Omega \implies C_5 = 10\text{nF}$
- Since each stage has a pure gain of 1, the overall gain is 1, which matches what we need; moreover, since we used an odd number of stages, we also matched the negative sign
- It's good to draw out the circuit at the very end to check for problematic open circuits and shorts, or extreme gains or attenuations