Lecture 3, Sep 27, 2023

Mortgages and Bonds

Mortgage Loans

- *Mortgages* are loans secured by real property – the money is borrowed with real property as collateral; upon default, the lender has the right to take the property
- Mortgage terminology:
  - Principal: the amount of money borrowed
  - Loan-to-value ratio (LTV): ratio of the mortgage loan to the value of the property
  - Mortgage rate: interest rate of the mortgage loan
  - Amortization period: agreed time horizon for payment
  - Term: an amount of time when the interest rate is fixed, usually shorter than the amortization period
    * After a term, the monthly payments may change as a result of interest rate changes
    * Note that even though the term is the length of the agreement, monthly payments are calculated based on the amortization period
- Example: $500,000 home with a 20% down payment, 5% mortgage rate, 5 year payment, 25 year amortization period
  - What is your monthly payment for the first term?
    * $i = \frac{5\%}{12}, N = 25 \cdot 12$
    * $A = 400,000(A/P, i, N) = 2338$
  - At the end of the term, the interest changes to 6%, what is the new monthly payment?
    * First we need to calculate the amount owing:
      - With interest the principal would have grown to: $400,000(F/P, i, 5 \cdot 12)$
      - We paid: $2338(F/A, i, 5 \cdot 12)$
      - Therefore we still owe $354,320$
    * Now we can calculate the monthly payment as $A = 354,320(A/P, \frac{6\%}{12}, 20 \cdot 12) = 2538$
    * Note that this amount is higher than the first set because of the increase in interest rate
- In real life the interest can vary within a term, you can make additional payments, etc
- Example: on a principal of $800k, 5 years ago the interest was 1.5% with monthly payments, with a term of 5 years and an amortization of 25 years; the new interest is now 5%, what are the new monthly payments?
  - $800000 = A(P/A, 0.1246\%, 25 \times 12, 25 \cdot 12) \implies A = 3197.74$
  - $F = 800000(F/P, 0.1246\%, 5 \cdot 12) - 3197.24(F/A, 0.1246\%, 5 \cdot 12) = 662975$
  - $A = 992975(A/P, 0.4124\$, 20) = 4356.58$

Bonds

- Bonds are financial instruments issued by firms and governments to raise funds to finance projects, as a special form of long-term loan
- The creditor promises to pay a stated amount at specified intervals for a defined period (*coupon payments*) and the final amount at a specific date (*face value* at the *date of maturity*)
  - Bonds are often tradeable at any given time, so the payment may not always go to the person who initially held the bond
- Bonds have a *coupon rate* which is used to calculate coupon payments (i.e. interest)
  - e.g. a $1000 bond at 9.25% interest with 2 payments per year pays $9.25\% \cdot 1000 \cdot \frac{1}{2} = 46.25$ per payment
- Example: What is the present worth of a $1000 bond that matures in 10 years, with 9.25% coupon rate paid every 6 months?
  - Suppose the bank is paying 10% interest; how much do we need to put into the bank now to get this cash flow?

  - $P = 46.25(P/A, \frac{10\%}{2}, 10 \cdot 2) + 1000(P/F, \frac{10\%}{2}, 10 \cdot 2) = 953$
- Since there is a risk of default, the creditor must demand a lower price on the bond to compensate for the risk; this equates to a higher interest rate
  - Example: if we are only willing to pay $700 for the aforementioned bond, what is the interest rate?
    * $700 = 46.25(P/A, \frac{i}{2}, 20) + 1000(P/F, \frac{i}{2}, 20)$
    * We can solve this to get $i = 15.2\%$
  - This new higher interest rate is called the *yield* – it is determined by the market and not the bond issuer
  - The higher the yield, the cheaper the bond will be at the present moment; if the risk suddenly increases, yield will also increase and the present value of the bond will decrease
  - Another way to interpret the yield is how much you prefer getting paid at the present moment than in the future
  - Yield rates are expressed as annual rates based on compounding equivalent to the coupon payment frequency
    * If the compounding period is less than a year, the interest is simply multiplied to get the annual interest (unless effective interest is stated)
- Yield is determined by the market expectation of risk and inflation
  - The more stable a country's economy, the higher the yield
- Longer time to maturity also increases the yield, since the risk increases as we have to wait longer to get the money
- Example: A $10000 bond was bought that will mature in 8 years, with 12% coupon payments paid quarterly; if the yield is 10%, how much is the present value of the bond?
  - Coupon payment is $10000 \cdot \frac{12\%}{4} = 300$
  - $P = 300(P/A, \frac{10\%}{4}, 4 \cdot 8) + 10000(P/F, \frac{10\%}{4}, 4 \cdot 8) = 11092$
- Example: A $880 bond maturing in 2 years pays $45 coupon every 6 months was bought for $1000; what is the effective annual yield?
  - $1000 = 45(P/A, i^*, 4) + 880(P/F, i^*, 4)$
  - To solve this we can guess and interpolate; we will get 1.57% yield per 6 months
  - Therefore $i_e = (1 + i^*)^2 - 1 = 3.17\%$
- Note that calculating the yield rate usually involves solving an equation that is not analytically solvable
  - We can use a computer or guess-and-interpolate by hand
  - To guess an appropriate yield rate, note that if the bond sells for less than its face value, then the yield rate is higher than the coupon rate; if the bond sells for higher than its face value, then the yield rate is lower than the coupon rate
  - Try finding a yield rate that gives a present value higher than the actual price, and another one that is lower than the actual price, so to find the actual yield rate we can interpolate
  - Extrapolation is acceptable in a test