Lecture 1, Sep 13, 2023

Interest Rates

- "Time Value of Money": borrowing money has a cost for the borrower; lending money should create value for the lender (investor/creditor)
- Factors impacting interest rates:
  - *Inflation*: higher expected inflation leads to increase in interest rates
  - *Credit (default) risk*: higher risk of the borrower not being able to pay back leads to higher interest rates
  - *Liquidity risk*: higher liquidity risk (not being able to access the invested funds during the investment) leads to higher interest rate
  - *Maturity risk*: longer term investments (longer maturity) increase other risks, leading to higher interest rates
- The interest might only be paid yearly, but the money is being made in real time!
- Two types of interest (assume compound unless stated otherwise):
  - *Simple*: interest applies only to original principal, e.g. GICs (Guaranteed Investment Certificates)
    - After N periods, amount owed is $P(1 + Ni)$ where P is the principal, i is the interest rate
  - *Compound*: interest applies to principal and all interest, e.g. mortgages, credit card debt
    - After N periods, amount owed is $P(1 + i)^N$
- We can also compound multiple times a year with a fraction of the interest; for m subperiods a year, the subperiod interest rate is $i_s = \dfrac{r}{m}$
  - At the end of the year we would have $F = P\left(1 + \dfrac{r}{m}\right)^m$ after all m subperiods
  - The more frequently you compound, the more interest you will earn, so to compare investments with different compound periods, we need an *effective interest rate*
- *Effective interest rate* (i_e): the equivalent interest rate of an account that is compounded just once over the stated time periods
  - $i_e = \left(1 + \dfrac{r}{m}\right)^m - 1$
  - As m increases, i_e increases and approaches a finite limit (known as *continuous compounding*)
  - For continuous compounding, $i_e = \lim\limits_{m\to\infty}\left(1 + \dfrac{r}{m}\right)^m = e^r - 1$
    - Note that the continuous interest rate has units of percent per time, while every other rate has units of percent (i.e. unitless)
- Example: given a mortgage rate of 5% (compounded semi-annually), how much interest will you pay on a \$100k mortgage in the first month?
  - $r_{s/s} = 2.5\%$ (semi-annual interest compounded semi-annually)
  - $(1 + 2.5\%) = (1 + r_{m/m})^6$ solve to get an $r_{m/m} = 0.4124\%$
  - Therefore the interest per month is $\$100000 \times 0.4124\% = \$412.4$

Present Value and Factor Notation

- The formulas can be reversed to calculate the present value: $PV = \dfrac{FV}{(1+i)^N}$
  - Money in the future is worth less, because you can't invest it; instead we calculate how much we would invest to get that amount in the future given some interest rate
  - In factor notation: $PV = FV(P/F, i, N) \implies (P/F, i, N) = \dfrac{1}{(1+i)^N}$
  - To calculate future value: $FV = PV(F/P, i, N) \implies (F/P, i, N) = (1 + i)^N$

Lecture 2, Sep 20, 2023

Cash-Flow Analysis

Cash-Flow Diagrams

- Categories of cash flows:
  - Start of project:
    * First (capital) cost: expense to build/buy/install
  - During the project:
    * Revenue cost: receipts from sales of products or services
    * Operation and maintenance (O&M) costs: expenses incurred on a regular basis (e.g. labour)
    * Overhaul: major expenditure that occurs partway through the life of an asset
  - At the end of the project:
    * Salvage value: money gained for sale or disposal of a product
    * Scrap value: value of the materials that the item is made of
    * Disposal costs: costs to dispose of the item
  - These are collectively referred to as *project life-cycle costs*
- A cash-flow diagram summarizes the timing and magnitude of cash-flows using arrows that go up/down
  - The x axis (which is discrete) shows the timing of cash flows and y axis shows their value
    * Down is usually cash outflow, up is inflow
    * Note the end of one period is the beginning of the next; a cash flow at the beginning of year 1 would be at time 0 (now)
      - Cash flows that occur during a period are assumed to occur at the end of the period
      - Interest is compounded once per period unless specified
  - Cash flows at the same time are summed

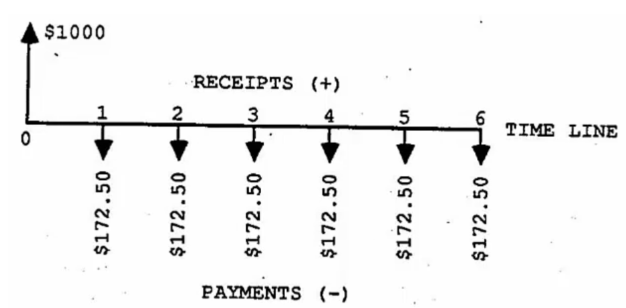

Figure 1: Example cash-flow diagram: Borrowing \$1000 from a bank and paying it back in 6 monthly installments of \$172.50 (1% interest per month)

- Cash-flows can be equivalent:
  - Mathematical equivalence: equivalence of cash-flows due to mathematical equivalence of time and money
    * Two cash-flows, P_t at time t and F_{t+N} at time $t+N$ are mathematically equivalent with respect to the interest rate i if $F_{t+N} = P_t(1+i)^N$
  - Market equivalence: a consequence of the ability to exchange one cash-flow for another at zero cost
    * Once can exchange cash-flows between present and future amounts; *borrowing* to exchange a future cash-flow for a present one, and *lending/investing* to give up a present cash-flow for a future one
    * Example: if a T-bill of \$1000 due to mature in 6 months is selling for \$990.10, then the "market" has agreed that the appropriate interest rate is 1% over 6 months
    * The cash flow of \$1000 6 months in the future is equivalent to a \$990.10 right now
  - Decisional equivalence: due to indifferences on the part of decision maker among available choices
    * For a decision maker, two cash-flows P_t, F_{t+N} are equivalent if they are indifferent about the two

    * The interest rate is not a priori information but rather implied (which can be calculated)
  - The market tells us what interest rate we should take based on the risk; mathematical equivalence allows us to use that interest rate to do the math; decisional equivalence allows us to use the math to decide what to take

Present Value of Cash-Flows

- There are 5 common types of regular cash flows:
  - Single payments (aka receipts): a single cash payment occurring at some time
  - Perpetuity: a cash flow of magnitude A occurring at regular intervals indefinitely
  - Annuity: like a perpetuity but only over N periods
  - Arithmetic gradient: like an annuity, but the amount increases by G each period (A the first time, $A+G$ the second time, $A+2G$ the third time and so on)
  - Geometric gradient: like the arithmetic gradient but the amount grows by a factor of $1+g$ each period
- Factor notation: to convert from cash flow Y to X, use the factor $X = Y(X/Y, i, N)$ where i is the interest rate and N is the time period
  - X and Y are P (present), F (future), A (annuity), G (arithmetic gradient) or Geom (geometric gradient)
  - For the geometric gradient we also need the growth rate, so $X = G(X/G, i, g, N)$
  - Some factors have special names:
    * F/P: compound amount factor
    * P/F: present worth factor
    * A/F: sinking fund factor
    * F/A: series compound amount factor
    * A/P: capital recovery factor
    * P/A: series present worth factor
  - Reversing X and Y simply inverts the factor
- P/F, F/P factors can be derived by simply applying interest: $(F/P, i, N) = (1+i)^N$
- $(P/A, i, N) = \dfrac{1}{i} - \dfrac{1}{i(1+i)^N}$
  - This can be derived through a series sum or by noting that a perpetuity of amount A has present value $\dfrac{A}{i}$, since the amount of interest per period is $A = Pi$
- For an arithmetic gradient, $P = A(P/A, i, N) + G(P/G, i, N)$ where $(P/G, i, N) = \dfrac{1}{i^2}\left(1 - \dfrac{1+iN}{(1+i)^N}\right)$
  - The first part, $A(P/A, i, N)$ is the value of the initial base annuity
  - The second part is the value of the growth
  - This can be derived by turning the arithmetic gradient into a series of annuities
  - $P = G\displaystyle\sum_{k=1}^{N-1} \frac{(P/A, i, N-k)}{(1+i)^k}$ ignoring the annuity starting on year 1
- $(P/Geom, i, g, N) = \dfrac{1}{1+g}(P/A, i^0, N)$ where $i^0 = \dfrac{1+i}{i-g} - 1$
  - Equivalently $(P/Geom, i, g, N) = \dfrac{1 - \left(\frac{1+g}{1+i}\right)^N}{i-g}$
  - Note for a geometric gradient cash flow, there is nothing at time 0, then at time 1 a cash flow of A, then at time 2 a cash flow of $A(1+g)$ and so on

Examples

- Consider annuity that pays \$10 per month for 1 year. If interest is 1% per month compounded monthly, what is the equivalent continuous payment right now?
  - $P = 10(P/A, 1\%, 12) = \$112.55$

- $P = \int_0^1 Ae^{-r_{cc}t}\,\mathrm{d}t$ and we can find $(1+1\%)^{12} = e^{r_{cc}} \implies r_{cc} = 11.94\%/\text{yr}$
- $\$112.55 = A\int_0^1 e^{-0.1194t}\,\mathrm{d}t \implies A = \$119.40\text{yr}^{-1}$
- Determine the effective annual rate for an investment that earns 15% per year based on quarterly compounding for the first 4 months, then 11% per year based on continuous compounding for another 5 months
  - $\left(1+\frac{15\%}{4}\right)^{\frac{4}{3}} e^{11\%\cdot\frac{5}{12}} = (1+r_{eff})^{\frac{9}{12}}$
- A property was sold for \$10M. The buyer wants a VTB (vendor-take-back) mortgage for \$5M, with 4% interest and \$500k of principal paid yearly for 10 years; what is the present value to the seller at 10% interest per year?
  - VTB is if the buyer defaults, the seller gets the building back
  - The buyer first gets \$5M in year 0, then \$500k each year plus some interest each year
    - As the year goes on the interest decreases, because the principal that the interest applies to decreases
    - In year 1 the interest is $\$5000000 \cdot 4\%$, in year 2 it's $\$4500000 \cdot 4\%$ and so on
    - This is an arithmetic gradient with a negative gradient
  - We have $A = 4\% \times \$5000000 = \$200000, G = -\$500000 \cdot 4\% = \$20000$
  - $P = 5000000 + (500000 + 200000)(P/A, 10\%, 10) - 20000(P/G, 10\%, 10) = 8843370$
    - Note we had to add 200k to the annuity value because of the declining interest

Lecture 3, Sep 27, 2023

Mortgages and Bonds

Mortgage Loans

- *Mortgages* are loans secured by real property – the money is borrowed with real property as collateral; upon default, the lender has the right to take the property
- Mortgage terminology:
  - Principal: the amount of money borrowed
  - Loan-to-value ratio (LTV): ratio of the mortgage loan to the value of the property
  - Mortgage rate: interest rate of the mortgage loan
  - Amortization period: agreed time horizon for payment
  - Term: an amount of time when the interest rate is fixed, usually shorter than the amortization period
    - After a term, the monthly payments may change as a result of interest rate changes
    - Note that even though the term is the length of the agreement, monthly payments are calculated based on the amortization period
- Example: \$500,000 home with a 20% down payment, 5% mortgage rate, 5 year payment, 25 year amortization period
  - What is your monthly payment for the first term?
    - $i = \frac{5\%}{12}, N = 25 \cdot 12$
    - $A = 400,000(A/P, i, N) = 2338$
  - At the end of the term, the interest changes to 6%, what is the new monthly payment?
    - First we need to calculate the amount owing:
      - With interest the principal would have grown to: $400,000(F/P, i, 5 \cdot 12)$
      - We paid: $2338(F/A, i, 5 \cdot 12)$
      - Therefore we still owe $354,320$
    - Now we can calculate the monthly payment as $A = 354,320(A/P, \frac{6\%}{12}, 20 \cdot 12) = 2538$
    - Note that this amount is higher than the first set because of the increase in interest rate
- In real life the interest can vary within a term, you can make additional payments, etc

- Example: on a principal of \$800k, 5 years ago the interest was 1.5% with monthly payments, with a term of 5 years and an amortization of 25 years; the new interest is now 5%, what are the new monthly payments?
  - $800000 = A(P/A, 0.1246\%, 25 \times 12, 25 \cdot 12) \implies A = 3197.74$
  - $F = 800000(F/P, 0.1246\%, 5 \cdot 12) - 3197.24(F/A, 0.1246\%, 5 \cdot 12) = 662975$
  - $A = 992975(A/P, 0.4124\$, 20) = 4356.58$

Bonds

- Bonds are financial instruments issued by firms and governments to raise funds to finance projects, as a special form of long-term loan
- The creditor promises to pay a stated amount at specified intervals for a defined period (*coupon payments*) and the final amount at a specific date (*face value* at the *date of maturity*)
  - Bonds are often tradeable at any given time, so the payment may not always go to the person who initially held the bond
- Bonds have a *coupon rate* which is used to calculate coupon payments (i.e. interest)
  - e.g. a \$1000 bond at 9.25% interest with 2 payments per year pays $9.25\% \cdot 1000 \cdot \frac{1}{2} = 46.25$ per payment
- Example: What is the present worth of a \$1000 bond that matures in 10 years, with 9.25% coupon rate paid every 6 months?
  - Suppose the bank is paying 10% interest; how much do we need to put into the bank now to get this cash flow?
  - $P = 46.25(P/A, \frac{10\%}{2}, 10 \cdot 2) + 1000(P/F, \frac{10\%}{2}, 10 \cdot 2) = 953$
- Since there is a risk of default, the creditor must demand a lower price on the bond to compensate for the risk; this equates to a higher interest rate
  - Example: if we are only willing to pay \$700 for the aforementioned bond, what is the interest rate?
    * $700 = 46.25(P/A, \frac{i}{2}, 20) + 1000(P/F, \frac{i}{2}, 20)$
    * We can solve this to get $i = 15.2\%$
  - This new higher interest rate is called the *yield* – it is determined by the market and not the bond issuer
  - The higher the yield, the cheaper the bond will be at the present moment; if the risk suddenly increases, yield will also increase and the present value of the bond will decrease
  - Another way to interpret the yield is how much you prefer getting paid at the present moment than in the future
  - Yield rates are expressed as annual rates based on compounding equivalent to the coupon payment frequency
    * If the compounding period is less than a year, the interest is simply multiplied to get the annual interest (unless effective interest is stated)
- Yield is determined by the market expectation of risk and inflation
  - The more stable a country's economy, the higher the yield
- Longer time to maturity also increases the yield, since the risk increases as we have to wait longer to get the money
- Example: A \$10000 bond was bought that will mature in 8 years, with 12% coupon payments paid quarterly; if the yield is 10%, how much is the present value of the bond?
  - Coupon payment is $10000 \cdot \frac{12\%}{4} = 300$
  - $P = 300(P/A, \frac{10\%}{4}, 4 \cdot 8) + 10000(P/F, \frac{10\%}{4}, 4 \cdot 8) = 11092$
- Example: A \$880 bond maturing in 2 years pays \$45 coupon every 6 months was bought for \$1000; what is the effective annual yield?
  - $1000 = 45(P/A, i^*, 4) + 880(P/F, i^*, 4)$
  - To solve this we can guess and interpolate; we will get 1.57% yield per 6 months
  - Therefore $i_e = (1 + i^*)^2 - 1 = 3.17\%$

- Note that calculating the yield rate usually involves solving an equation that is not analytically solvable
  - We can use a computer or guess-and-interpolate by hand
  - To guess an appropriate yield rate, note that if the bond sells for less than its face value, then the yield rate is higher than the coupon rate; if the bond sells for higher than its face value, then the yield rate is lower than the coupon rate
  - Try finding a yield rate that gives a present value higher than the actual price, and another one that is lower than the actual price, so to find the actual yield rate we can interpolate
  - Extrapolation is acceptable in a test

Lecture 4, Oct 4, 2023

Capital Asset Pricing Model (CAPM)

- Valuation is the analytical process of determining the current or projected worth of an asset or a company
  - Performing a valuation requires projected cash-flows and an interest rate determined by risk
  - Data from "the market" (i.e. the stock market) is used to price risk
- Financial risk is defined as the uncertainty in a future payoff
- Modern portfolio theory (MPT) provides a mathematical framework for assembling a portfolio of assets to maximize return for a given risk level
  - The risk and return characteristics of an investment should not be viewed alone, but evaluated by how it affects the overall portfolio

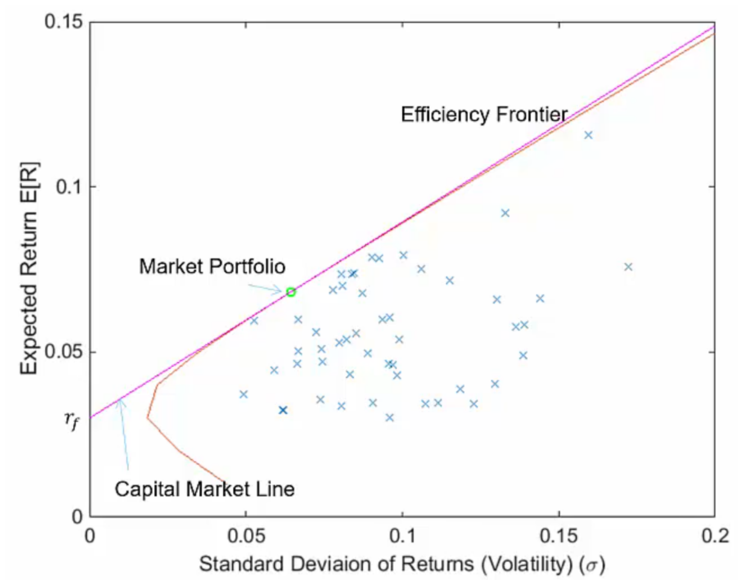

Figure 2: Expected return vs. volatility.

- Consider a portfolio of stocks; define the return vector for the ith stock as $\vec{R}_i = \begin{bmatrix} r_{t_1} \\ r_{t_2} \\ \vdots \\ r_{t_m} \end{bmatrix}$ where r_{t_j} is the (continuous) return of that stock at time t_j
  - Define the volatility σ such that $\sigma_i^2 = \mathrm{var}(\vec{R}_i)$, which we can calculate using past historical market data
  - If we assume that we can invest some fraction x_i of our total capital in stock i, then we can try to maximize the expected return $E[\vec{R}_p]$ while trying to minimize risk σ_p

  - Optimizing this gives us an *efficiency frontier* (line in yellow); assuming that we can invest/borrow at the risk-free rate, we can draw a line from the risk-free rate that is tangent to the efficiency frontier; the tangent point is known as the *market portfolio* or *tangency portfolio*
    * The market portfolio is a theoretical portfolio that contains every single investment in existence, with each one weighted by its value; mathematically, this maximizes the return for a given volatility
    * Since the efficiency frontier is concave down, the market portfolio and the risk-free rate are the theoretically best investments given a certain level of risk
    * Thus if we assume all investors act perfectly rationally, everyone will try to invest at either the market portfolio or interest-free rate
    * Based on the proportion of investments in each of the two categories, we can calculate the overall return rate; an investor can also take on debt at the risk-free rate and invest it at the market portfolio which increases both risk and expected return, moving up the tangent line
  - This is Capital Asset Pricing Model (CAPM)
- Note that CAPM is an ideal mathematical model for something driven by investor behaviour, so it has to make some assumptions
  - Correlations between assets are fixed and constant forever (i.e. volatilities in stocks don't change and neither do their correlations)
  - All investors act perfectly rationally to maximize their economic utility
  - All investors have access to the same information at the same time
  - Investors have an accurate conception of possible returns (i.e. the probability beliefs of investors match the true distribution of returns)
  - No taxes or transaction costs
  - Investor behaviour do not influence stock prices
  - Any investor can lend and borrow an infinite amount at the risk-free interest rate
  - All securities can be divided into parcels of any size
- CAPM gives the expected return for asset i as $E[R_i] = r_f + \beta_i(E[R_{MP}] - r_f)$ where r_f is the risk-free rate and $\beta_i = \frac{\sigma_{i,MP}}{\sigma_{MP}^2} = \frac{\rho_{i,MP}\sigma_i}{\sigma_{MP}}$
  - β is a measure of risk for the company, relative to the market as a whole
    * $\beta > 1$ means the company is more risky than the market, and $0 < \beta < 1$ means the company is less risky than the market
    * $\beta \leq 0$ is almost never seen
  - Theoretically if we plotted the return against β for every single stock, it will lie on a perfect line
    * In reality if you did this it would not end well
  - $E[R_{MP}] - r_f$ is known as the *market premium* or *risk premium*, which is the difference in rate of return between the market portfolio and risk-free rate
  - We are modeling the company return as $R_{C,t} = \alpha_C + \beta_C R_{MP,t} + \varepsilon_{C,t}$
  - Volatility in $R_{MP,t}$ represents a systematic, or market risk, that is present throughout the entire market
  - Volatility in $\varepsilon_{C,t}$ represents an idiosyncratic, or firm-specific risk
  - The idea is that $E[\varepsilon_{C,t}] = 0$, so by diversifying their investments enough, a investor should be able to completely eliminate idiosyncratic risk
    * Therefore idiosyncratic risk has no inherent value, so investors should only be rewarded for market risk
- Any company can be broken down into 3 parts: assets, equity, and debt
  - The return on equity R_E comes from CAPM
  - The return on assets is a weighted sum $R_A = \frac{E}{E+D}R_E + \frac{D}{E+D}R_D$

Arbitrage

- In general arbitrage is the idea of taking advantage of inconsistencies in prices to be able to pay a lower price than what you should have
  - In financial markets arbitrage is the practice of taking advantage of a price difference between two

or more markets
  - In academic use, it refers to the opportunity to achieve a risk-free gain at a rate greater than the risk-free rate
    * A fundamental assumption is that there is no true arbitrage, or the "no free lunch" assumption
  - Statistical arbitrage occurs when a gain is achieved at a higher risk than one should for a given level of risk
- Example: if you can buy a bushel of corn for \$10 today and enter into a forward contract to deliver the corn for a pre-settled price of \$12 per bushel one year from now, and it costs you nothing to store the corn; say you could borrow at 10%, then this is an arbitrage opportunity, because we could just borrow money to buy corn and then sell it a year later to get free money at no risk
  - However in the real world, as demand for corn increases the price will also increase
  - Assuming the forward price of \$12 does not change, the eventually the price right now would increase to \$10.91 so that we end up with no net gain
  - This is an example of a *future* or *forward contract*, which locks in a specific price for an asset; it is used by companies or investors to hedge against risks or speculate
    * A forward contract is an obligation to buy or sell at a specific price and time, and typically not traded
      - Sellers and buyers are involved in a forward transaction, and are both obligated to fulfill their end of the contract at maturity
    * Futures are similar but they are settled daily (not just at maturity), so they can be bought and sold at any time and are traded on exchanges
    * For this course we don't care about the difference
- In general to use the no-arbitrage principle to price an investment, we find two alternative investment paths and conclude that they must bring equal returns
- Example: given yields r_{t_1}, r_{t_2} at two times t_1, t_2, calculate the appropriate forward rate that an investor could lock into today from t_1 to t_2
  - Suppose we invest P_0 today, then at t_1 we have $F_1 = P_0 e^{r_{0,1} t_1}$, or at t_2 we have $F_2 = P_0 e^{r_{0,2} t_2}$
  - If we take our money from t_1 and reinvest it, we have $F_2' = F_1 e^{r_{1,2}(t_2 - t_1)}$
  - The no arbitrage principle says that $F_2 = F_2'$, otherwise there would be an arbitrage opportunity
  - Therefore we have $P_0 e^{r_{0,2} t_2} = P_0 e^{r_{0,1} t_1} e^{r_{1,2}(t_2 - t_1)} \implies r_{1,2} = \dfrac{r_{0,2} t_2 - r_{0,1} t_1}{t_2 - t_1}$
- Another strategy is to build a portfolio with the same risk as the investment, then by no-arbitrage the worth of this portfolio is the same as the original investment
- Example: a project will have a \$140 payoff if the market goes up or a \$30 payoff if the market goes down; the risk-free rate is 5%, the market portfolio is priced at \$100 and will be worth \$120 if the market goes up, or \$95 if the market goes down; what is the valuation of the project?
  - We want to replicate a portfolio so that it has the same risk/payoff as the project
  - We will buy a units of the market portfolio and b units of the risk-free investment
    * Note a and b could be negative, in which case we would be borrowing
  - In the up-market case then our payoff would be $120a + 1.05b = 140$; in the down-market case the payoff is $95a + 1.05b = 30$
    * Note that in both cases the risk-free payoff is 1.05 since regardless of market the interest is still 5%
    * Using this we can solve to get $a = 4.4, b = -369.53$
  - Therefore the project is worth $100a + 1b = 70.48$
    * Note that we did not use any probability of the market going up or down at this point
  - If the probability if the market going up is 60%, what is the expected return on the MP and project?
    * For the MP the expected payoff is $120 \cdot 0.6 + 95 \cdot 40 = 110$, so the return is $\dfrac{110}{100} - 1 = 10\%$
    * For the project it is $\dfrac{140 \cdot 0.6 + 30 \cdot 0.4}{70.48} - 1 = 36.2\%$
    * Notice that the project has much higher return, because its volatility is much greater
  - What if the expected payoff was \$140 regardless of the market?

      * This means the project is now risk-free, so we discount by the risk-free rate $\frac{140}{1.05} = 133$

Lecture 5, Oct 11, 2023

Project Comparisons

- In engineering economics we are concerned with the valuation of "projects", or investments; we want to evaluate and compare different options when there are multiple solutions or opportunities presented
- Projects can have 3 different types of relations:
  - Independent: expected costs and benefits of each project do not depend on whether or not the other project is chosen
    * In this case we consider each project entirely separately
  - Mutually exclusive: when selecting one project automatically exclusives all other projects, so you can only select one at a time
    * In this case we find the best option
  - Related but not mutually exclusive: when you can select more than one option, but selecting one may affect the selection of another option
    * In this case we form mutually exclusive combinations of projects, and then find the best option
- A cash flow for a typical project involves a initial cost at time zero, and then over the lifetime of the project we will have some income revenue and some maintenance costs, and then at the end of the lifetime we will get salvage value
- To find the present value for such a cash flow, we need the discount rate – this is determined by the Minimum Acceptable Rate of Return (MARR)
  - MARR is based on risk and expected rate of return of the best alternative
    * MARR generally lies between 10% to 30% but varies between firms
    * The firm see this closely tied to their cost of capital, or WACC; typically MARR is greater than or equal to the WACC
    * MARR represents an opportunity cost, since investing in one project means giving up another
  - Also known as the *hurdle rate*, because projects that earn less than MARR are not acceptable
  - We will assume that money can always be invested at MARR, i.e. doing nothing will earn money at MARR
- The weighted average cost of capital is (without taxes) $R_{WACC} = R_A = \frac{E}{E+D}R_E + \frac{D}{E+D}R_D$, where R_A is the return on assets, R_E is the expected return on equity and R_D is the expected return on debt
  - WACC represents how much investors expect the company to grow – if the rate of return is greater than WACC, then we'll be making enough money to pay off the shareholders and creditors; this is why we typically have MARR higher than WACC
  - The value of a firm is split into two parts: debt (money lent to the firm) and equity ("owners" stake in the firm)
  - Debt has less risk so less return; equity has more risk so more return
    * When a company goes bankrupt, the bondholders may still get money back (liquidating assets) but shareholders get nothing
  - The rates used here are market rates, which can be determined through CAPM
- *Present Worth, PW* (also known as Net Present Value, NPV) is the present value of benefits minus costs, discounted at MARR
  - This is the amount by which this project is beating the best alternative expressed in today's value
  - Projects with a positive PW are acceptable, while a negative PW is unacceptable
  - Note a negative PW does not mean losing money, but that it's worse than doing nothing
  - Example: purchasing a Honda Civic with a life of 10 years, 5% MARR; annual benefits worth \$4000, annual costs worth \$1000, initial cost is \$20000, salvage cost is \$4000
    * $PW = -20000 + (4000 - 1000)(P/A, 5\%, 10) + 4000(P/F, 5\%, 10) = 5621$
- *Annual Worth, AW* is the equivalent annuity of the PW
  - With the same example as above, $AW = -20000(A/P, 5\%, 10) + 3000 + 4000(A/F, 5\%, 10) = 728$
- Note that if present worth is positive, then so will the future and annual worths; so if a project is a go

with PW, it will always be a go with AW and FW

- Example: Mutually exclusive projects: compare the Honda Civic above to a Tesla, costing \$100000 with \$12000 annual benefits, 10 years of life, \$14000 salvage value, and the same MARR
  - Since both cars have a 10 year life, we will choose this as the time horizon
  - For the Tesla this is $PW = -100000 + 12000(P/A, 5\%, 10) + 14000(P/F, 5\%, 10) = 1255$
  - Since the Honda has higher PW, we should choose it over the Tesla
- What if the two projects have different time horizons?
  - Repeated lives: assume that the project repeats itself, i.e. once the project ends, we can replicate it identically and do it again
    * Use the least common multiple of the individual projects; e.g. for a 4 year project and a 5 year project, use a 20-year time horizon
    * Example: compare the Honda to a Lexus costing \$40000, with \$8000 annual benefits and \$13000 salvage value, but only 5 years of life
      - For 5 years: $PW = -40000 + 8000(P/A, 5\%, 5) + 13000(P/F, 5\%, 5) = 4822$
      - For 10 years: $PW = 4822 + 4822(P/F, 5\%, 5) = 8600$
      - Therefore the Lexus is a better choice
  - Use annual worth
    * This is equivalent to comparing the present worth of repeated projects
    * Previous example with AW:
      - For Lexus, $AW = -40000(A/P, 5\%, 5) + 8000 + 13000(A/F, 5\%, 5) = 1114$
      - This is bigger than the \$728 of the Honda, so we make the same decision
  - Comparison with repeated lives and annual worth are mathematically equivalent, since a repeating cash flow has the same annuity has a non-repeating cash flow
  - Study period: adopt a specified time period for comparison; if a project terminates after the study period, assume we can terminate it early and adjust the salvage value as necessary
    * Previous example, but assume that at 5 years, the Honda has a salvage value of \$7000
      - For the Honda now, in 5 years $PW = -20000 + 3000(P/A, 5\%, 5) + 7000(P/F, 5\%, 5) = -1527$
      - Therefore we should choose the Lexus

Lecture 6, Oct 18, 2023

Internal Rate of Return (IRR)

- A rate of return analysis starts by asking what is the rate of return on an investment
- The *internal rate of return* (IRR) is the interest rate (discount) that makes the present worth of benefits equal to the present worth of costs/investments
  - i.e. the IRR is the discount rate that make the total present worth of the project 0
  - A more profitable project has higher IRR, since higher benefits in the future need to be discounted more to match the same costs in the present
- IRR is often difficult to solve for analytically, so we will numerically solve or interpolate
  - If the cash flows near the present time are negative (simple investment), then increasing interest rate will lower the present worth
- In Excel this can be found using `NPV(i, cash flows)` to find the present worth, and using `IRR(cash flows, guess)` to find the IRR
  - Note that NPV assumes the cash flow starts on year 1!
- For simple investments (i.e. negative cash flows occur before positive ones), we want IRR greater than MARR for the project to be worthwhile
  - IRR is a useful measurement of profitability in these scenarios
- For non-simple investments, where cash flow changes signs more than once, the present worth is no longer monotonically increasing/decreasing with respect to discount rate, so we may get multiple possible IRRs that cause the present worth to be zero
  - This is a limitation of using IRR
  - In these situations we can use multiple IRR

Payback Period

- The *payback period* is the time it takes for an investment to be recouped
  - For the regular payback period, revenue is summed until it is equal to the initial investment
  - The *discounted payback period* also takes into account an interest rate
- If we reach the initial investment partway between years, we can interpolate
- Note that this is not a sound method – it only pays attention to the time when costs are recovered, so benefits after the payback periods are ignored

IRR for Mutually Exclusive Projects

- IRR ignores the size of the investments
- For an investment, the money not invested is assumed to be invested at MARR
- An investment with lower IRR can give you higher overall returns if the project is larger, so you can invest more money, or if the project is longer term
- Incremental IRR compares incremental differences in present and annual worth between two projects
- Incremental analysis steps:
  1. Order investments by first cost in increasing order
  2. Start with the "do nothing" option
  3. Evaluate the current option against the previous best option based on IRR
     - e.g. if this project has a first cost that's \$100 more expensive but gives an annual worth of \$25 per year, then the incremental IRR can be obtained by $PW = -100 + 25(P/A, i^*, 10) \implies i^* = 21\%$
     - If the incremental IRR is lower than MARR, then it is not worth it to switch projects; this means that the extra money is best invested as MARR

Lecture 7, Oct 25, 2023

Depreciation

- Assets starts to lose value as soon as they're purchased – this is the process of *depreciation*:
  - Use-related physical loss (wear and tear)
  - Time-related physical loss (things deteriorate over time even without use)
  - Functionality related loss (assets become obsolete over time, even without any physical changes)
- Depreciation can refer to the loss in value of the actual asset, or in our bookkeeping of the value of that asset (accounting)
- Assets are purchased for some price and at a later time, sold for a salvage value; the difference between these two is the depreciation
  - In accounting, we want to distribute this loss in value over time in a sensible manner
  - Depreciation is not an actual cash flow – it is an "accounting fiction" which does not directly enter into our cash flow calculations
  - However, depreciation is important to account for because it allows us to keep track of the price of assets at any time
- We can define value in a number of ways:
  - Cost basis: the value against which depreciation is measured (based on first costs)
  - Market value: the actual value of an asset if it were sold (usually hard to measure)
  - Book value: the value calculated for accounting purposes according to an agreed upon model
- Define B as the basis (original cost), BV_t as the book value at time t, N as the depreciable life of the asset, SA as the salvage value and D_t as the loss of value assigned to accounting period t
  - $BV_0 = B$, i.e. book value in the beginning is the basis
  - $BV_t = BV_{t-1} - D_t = B - \sum_{k=1}^{t} D_k$
- A number of methods can be used to calculate depreciation:
  - Straight line: simple linear decrease in value, i.e. constant loss in value each year

      * $D_t = \dfrac{B-S}{N}$ for all t
      * $BV_t = B - t\dfrac{B-S}{N}$
  - Declining balance: asset loses a fixed proportion of its value each year, i.e. exponential decay
      * $D_t = BV_{t-1}d$ where d is some proportion of depreciation
      * $BV_t = BV_{t-1}(1-d) = B(1-d)^t$
      * Given salvage value, we can calculate $d = 1 - \left(\dfrac{S}{B}\right)^{\frac{1}{N}}$
  - Double declining balance: setting $d = \dfrac{2}{N}$ and using the declining balance method as usual
  - Sum of the years' digits (SOYD): splits up depreciation so that year 1 depreciates N times some amount, year 2 depreciates $N-1$ times some amount, and so on until year N depreciating 1 times that amount
      * $SOYD = \displaystyle\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$
      * $D_t = \dfrac{N-t+1}{SOYD}(B-S)$
      * Faster than straight line during later years, slower than straight line during later years
      * This basically gives $\dfrac{N}{SOYD}(B-S)$ depreciation in the first year, $\dfrac{N-1}{SOYD}(B-S)$ in the second year, and so on, until $\dfrac{1}{SOYD}(B-S)$ depreciation in the final year
  - Units of production: assumes depreciation is a function of equipment use, rather than time
      * Depreciation is proportional to how much we use the asset
      * $D_t = \dfrac{\text{Production in year } t}{\text{Lifetime production}}(B-S)$
      * This requires us to know how many units we will be using each year
- Firms can use whatever method they choose to account for depreciation internally, while for tax purposes a model is specified by the government (usually declining balance)
- When the asset is sold at end of life, the market value may turn out to be different than the book value, in which case we need to make some adjustment
  - If the market value is lower, we have a *loss on disposal*, i.e. a one-time depreciation
  - If the market value is higher, we have a *recaptured depreciation*, i.e. a one-time negative deprecation
  - If the market value is higher than the cost basis, we have both a recaptured depreciation (difference between cost basis and book value, which brings us back to cost basis), and a *capital gain* (difference between market value and cost basis, which is the amount of money we make)
- We may also need to adjust the depreciation model partway through the asset's life; in this case we start at the current year and use the book value as the cost basis (but we do not change what we've already recorded)
- Example: asset purchased for \$900 and a declining balance depreciation of 40%; however at year 5 we find that it has no salvage value
  - We can reconcile this by setting the depreciation for year 5 only to match the salvage value
      * At year 4 the book value was \$116.64, so we would record the year 5 depreciation as \$116.64
- Example: asset purchased for \$1500 expected to last 15 years with no salvage value, but after 6 years we realize it will only last 4 more years (i.e. 10 years total); using straight line depreciation, what is the book value at the end of year 8?
  - $D_t = 100$ initially, so by year 6 we will have a book value of \$900
  - At this point we re-evaluate, $D_t = \dfrac{900-0}{4} = 225$
  - $BV_8 = 900 - 2 \cdot 225 = 450$

Accounting

- Assets, liabilities (book debt), and equity make up a company
  - Note even though book debt and market debt are similar, market equity and book equity can be

very different

- To get from one year's balance sheet to the next, we have an income statement – the balance sheets and the income statement describes everything that the company did
  - The income statement is revenue minus expenses, which gives net income
  - The net income minus dividends is added to the equity – note that this is in money per time period, i.e. a rate
- A *debit* is a positive transaction on the left hand side (assets, expenses); a *credit* is a positive transaction on the right hand side (liabilities, equity, revenue)
  - For every credit there is a debit and vice versa – the balance sheet is always balanced
- For every transaction we typically have a date and some amount
  - For cash transactions, we can directly determine one of the accounts to be our cash account
  - Examples:
    * If someone put $10 into the business, we debit cash $10 (assets, left hand side), and credit this person $10 in equity (right hand side)
    * If we sold something for $500, we debit cash $500 and credit revenue $500, but we might also have to debit some expenses and credit some wages payable

Lecture 8, Nov 1, 2023

Introduction to Accounting

- Two broad categories of accounting:
  - Managerial accounting: focused on internal use for decision makers, future oriented
  - Financial accounting: focused on external use by investors and government, typically must conform to standards
- Businesses are organized into 3 types:
  - Proprietorship: single person owns the entire business, who takes all liability
  - Partnership: two or more people own the business, who share the liability
  - Corporation: shareholders own the business, run by a board of directors elected by shareholders
- Accountants follow *generally accepted accounting principles*, or GAAP, a set of professional guidelines
- Main accounting principles:
  1. Entity concept
  2. Relevance characteristic
  3. Reliability/objectivity principle
  4. Cost principle
  5. Going-concern concept
  6. Stable monetary unit concept
- A *financial statement* is financial information about a business prepared in a systematic format; consists of:
  - Income statement: presents sales and expenses over time
  - Balance sheet: provides a snapshot summary of the company's account
    * The income statement takes us from the balance sheet from one point in time to another
    * Balance sheet units are dollars, income statement units are dollars per time
  - Statement of cash flow: summarizes the company's use of cash (can be derived from the income statement and balance sheet)
  - Statement of retained earnings: information about a firm's retained earnings, net income, amount distributed to stockholders
- There are 5 primary *accounts*:
  - On the balance sheet: assets = liabilities + equity
    * These are permanent
    * Assets: things the business has that it uses to make money
      - Short term current assets, e.g. cash, accounts receivable, inventory, etc
      - Long term assets, e.g. land, buildings, equipment
    * Liabilities: debts of the company

        - e.g. bank loans, notes payable, accounts payable, etc
        - Current liabilities are ones due within a year, long-term liabilities are due more than a year out
      * Equity: owners' claims to the assets of a corporation
        - Includes historic money invested and retained earnings
  - On the income statement: net income = revenues - expenses
    * These are temporary, closed at the end of a period and transferred to the balance sheet
- During each period, we take the retained earnings from the start, add the net income, subtract dividends, and get the ending balance of retained earnings
- *Transactions* are events that affect the accounts
- *Accrual accounting* means to record transaction when they occur, not when the cash is actually exchanged; this is opposed to *cash basis* accounting, where transactions are recorded when the cash is exchanged
  - Generally the first is much more useful to us

Double-Entry Accounting

- Each transaction has associated with it a *debit* and a *credit*
- Some accounts are "left hand side" accounts while some are "right hand side" accounts
  - Debits are increases to the left hand side while credits are increase to the right hand side
- Assets are on the left, while liabilities and equity are on the right
  - The retained earnings account (equity) increases with revenue, so revenue is considered a right hand side account
  - Expenses are considered a left hand side account
- Example: an owner of the business invests \$50000 cash in the business
  - This cash goes into assets, so we increase assets by \$50000; assets is a left hand side account so this is a debit
  - The associated credit is an increase in the common shares account, which is an equity account (right hand side)

Financial Ratios

- Financial ratios are a quick and dirty way for us to value a business
- These can be used as metrics to compare similar companies or against the industry average, typically within the same industry
  - Cross industry comparison is typically not done
- There are 5 types:
  1. Liquidity ratios: ability to pay current liabilities
     - Defined as current assets divided by current liabilities
     - Higher numbers are better, and a ratio below 1 could be a sign of distress
     - The acid-test ratio shows the ability to pay liabilities if they are due immediately
       * This is the ratio of (cash + short-term investments + net current receivables) to liabilities
  2. Efficiency ratios: ability to sell inventory and collect receivables
     - The inventory turnover ratio is the cost of goods sold to average inventory
     - This is a measure of the number of times that we would sell the average inventory in a given year
       * Note that if we change the time period in question, we have to adjust accordingly
     - Larger companies are generally less efficient on this front – lower inventory turnover ratio means more money is tied up in inventory
     - We can also measure this by days' inventory, the ratio of average inventory to average cost of goods sold in a day
       * This is the inverse of inventory turnover multiplied by the number of days in the period being analyzed
     - Accounts receivable turnover is the ratio of net credit sales to the average net accounts payable

- Days' receivables is the ratio of average receivables to average sales per day
  - This is on average how many days it takes to receive money from customers

3. Leverage ratios: ability to pay long-term debt
   - The debt ratio is the ratio of total liabilities to total assets (the proportion of assets financed with debt)
   - Times-interest-earned is the ratio of income from operations (sometimes EBIT) to the interest expense in the same period
     - This is how many times our income can cover the interest expense
4. Profitability ratios
   - The profit margin is the ratio of net income to net sales (i.e. how much profit we make off of every dollar in sales)
   - Return on assets (ROA) measures how profitably the company uses its assets; usually calculated as net income to assets
     - This is the ratio of income to how much was invested in the company
     - Sometimes calculated as (net income + interest times 1 minus taxes) over assets
   - Return on equity (ROE) is ROA but uses equity instead of all assets
   - Earnings per share (EPS) is the amount of profit on a per-share basis; this is the ratio of net income to number of shares
5. Performance ratios: analysis of shares as an investment
   - The price to earnings (P/E) ratio is the ratio of share price to earnings per share
     - A high P/E ratio could mean that the stock is overvalued, or that high growth is expected
     - This is not used for companies with zero or negative earnings
     - Note that a company with more debt would have a higher P/E ratio than an equivalent company with less debt
   - Dividend yield is the ratio of dividend per share to price per share
     - Mature and stable companies tend to pay dividends, while high growth companies tend to not pay dividends and instead reinvest the money
   - Market capitalization is the total market value of a company's outstanding shares of stock (not a ratio)
     - Companies are categorized by size by market capitalization: large-cap (10B+), mid-cap (2B to 10B), small-cap (300M to 2B)

Name	Expression	Interpretation	Notes
Current	$\frac{\text{curr. assets}}{\text{curr. liab.}}$	Ability to pay current liabilities	Below 1 could be a sign of distress
Acid Test	$\frac{\text{cash} + \text{curr. receivables} + \text{s.t. investments}}{\text{curr. liabilities}}$	Ability to pay liabilities if due immediately	Better if > 1
Inventory Turnover	$\frac{\text{COGS}}{\text{avg. inventory}}$	How many times per time period the average inventory is sold	Higher is more efficient; tends to be smaller for larger companies
Days' Inventory	$\frac{\text{avg. inventory}}{\text{avg. COGS per day}}$	Days to sell average inventory	Lower is more efficient
Acc. Receivable Turnover	$\frac{\text{net credit sales}}{\text{net acc. payable}}$	Number of times credit is collected	Higher is more efficient
Days' Receivables	$\frac{\text{avg. acc. receivables}}{\text{avg. sales per day}}$	How many days it takes to collect credit	Lower is more efficient
Days' Payables	$\frac{\text{avg. acc. payables}}{\text{avg. COGS per day}}$	How many days it takes to pay creditors	Higher means more funds are retained, too high might be problematic

Name	Expression	Interpretation	Notes
Debt Ratio	$\frac{\text{total liabilities}}{\text{total assets}}$	Proportion of assets financed with debt	Higher is more risky
Equity Ratio	$\frac{\text{total equity}}{\text{total assets}}$	Proportion of assets financed with equity	Higher is less risky
Times Interest Earned	$\frac{\text{income or EBIT}}{\text{interest expense}}$	How many times income can cover interest	Higher is better
Profit Margin	$\frac{\text{net income}}{\text{net sales}}$	How many dollars of profit per dollar of sales	Higher is better
Return on Assets (ROA)	$\frac{\text{net income}}{\text{total assets}}$	How profitably the company uses its assets	Higher is more efficient
Return on Equity (ROE)	$\frac{\text{net income}}{\text{total equity}}$	How profitably the company uses equity	Higher is more efficient
Earnings Per Share (EPS)	$\frac{\text{net income}}{\text{\# of shares}}$	Amount of profit per share	Higher is better
Price To Earnings (P/E)	$\frac{\text{share price}}{\text{EPS}}$	(Inverse) Relative value in shares	Higher means stock is overvalued or high growth
Dividend Yield	$\frac{\text{dividend per share}}{\text{EPS}}$	Dividends relative to stock price	Mature companies tend to pay more dividends
Market Capitalization	total market value of all shares	Measure of size of company	Large-cap (10B+), mid-cap (2-10B), small-cap (300M-2B)

Lecture 9, Nov 15, 2023

Taxation

- Companies are taxed based on taxable income, which revenues minus expenses
  - We will use a flat tax rate
- Revenues can include any income made by the company:
  - Sales revenues
  - Interest revenues
  - Capital gains (selling an asset above its book value)
    - In accounting, the true price of an asset is its book value, so the difference is counted as taxable income
- Expenses and deductions reduce the amount of taxable income:
  - Cost of goods sold
  - SG&A
  - Interest expenses
  - Depreciation expenses
  - Capital losses (selling an asset below its book value)
- Not all expenses are counted towards taxes, e.g. dividends, asset purchases and some others are not counted
  - Asset purchases are not counted because they still have value after you purchase them, so that money is not considered "consumed"

    - Therefore taxes don't affect first costs
  - However after purchase, depreciation and capital gains/losses do affect taxes
  - In general if something does not show up on the income statement (i.e. only balance sheet), it does not affect taxes
- Rate of returns are discounted by the tax rate for cash flows after tax, because we expect less profit
  - MARR and IRR after tax is the pre-tax rate multiplied by 1 minus the tax rate (as an approximation)
    - This gives the WACC
  - Note that equity rates (i.e. numbers from CAPM) are after-tax; so we don't need to adjust these
  - $R_{WACC} = \frac{E}{E+D}R_E + \frac{D}{E+D}R_D(1-t)$
    - Only the debt rate is discounted by tax

Capital Cost Allowance (CCA)

- Depreciation is not a real cash flow, but claiming depreciation saves on taxes
  - A company will want to depreciate as much as possible, as fast as possible due to time value of money
  - Government regulations exist to make sure companies can't arbitrarily depreciate
  - Note that on the actual balance sheet, a company might want to depreciate more slowly because this makes their income look better, but for tax purposes they must use CCA; this results in a discrepancy on the balance sheet
- CCA is the Canadian system for calculating depreciation and taxes
  - The capital cost allowance (CCA) is the amount of depreciation
  - The undepreciated capital cost (UCC) is the book value
  - The proceeds from disposition is the salvage value from selling the asset
- Assets are pooled into classes specified by the policy
  - All assets are pooled into classes, and the total value of all assets in each class is deprecated together at some specified rate
    - Assets are not depreciated individually
  - The depreciation method is similar to declining balance, but new assets purchased this year are only depreciated at half the rate
    - CCA = CCA rate times (new assets this year / 2 + UCC from last year)
    - This is known as the half-year rule
  - UCC this year is UCC from last year, plus new assets this year, minus CCA
- The amount of tax savings each year is the CCA from that year times the tax rate; this is recorded as a positive cash flow
- When selling the asset, the book value can be different from the salvage value
  - *Open book*: other items exit in the same CCA pool
    - Selling the asset does not close out the pool
    - Simply reduce the UCC of the pool by the salvage value
  - *Closed book*: no other items in the same pool; there are 3 cases:
    - *Terminal loss*: salvage value less than book value
      - Claim the loss, book value minus salvage value, which decreases taxable income
    - *Recapture*: salvage value greater than book value, but less than original price
      - Report the recapture, salvage value minus book value, which increases taxable income
    - *Capital gains*: salvage value greater than the original price
      - Report the recapture, original price minus book value, which increases taxable income
- Example: purchasing \$5000 worth of desks (class 8, CCA rate of 20%); what are the CCA and UCC for the first 4 years, assuming there is nothing in the same pool?
  - First year: \$5000 in purchases, no UCC from last year, CCA is 20% of \$5000 divided by 2 = \$500, so we're left with \$4500
  - Second year: \$0 in purchases, \$4500 from last year, CCA is 20% of \$4500 = \$900, so we're left with 3600

  - This gives us \$2304 at the end of the 4th year
  - In year 5, we sell the desks for \$1500; what is the gain/loss as a result of selling the desks, assuming open book?
    * CCA in year 5 is \$460
    * UCC after year 5 is \$2304, minus the CCA and minus the \$1500 sell value
    * Since this is open book, we have no direct gains or losses, so no direct tax implications
  - What if there are no other assets in class 8?
    * Since the UCC must go to 0, the book value for the desks would be \$2304 minus UCC, or \$1844
    * We sold it for a loss, so we can claim a loss of \$344, which is a tax saving
  - What if we sold it for \$2500?
    * This is greater than book value, so we have a recapture of \$2500 - \$1844 = \$656
    * This makes the taxes go up

Calculating Present Worth

- Using the explicit method:
  - Use the WACC (after-tax discounted MARR)
  - Revenues are discounted by the tax rate
  - No changes to first costs
  - Take into account depreciation tax savings every year by calculating CCA every year
  - Account for terminal losses or gains by comparing the final UCC to the salvage value, in a closed-book scenario
- The *tax benefit factor* τ is defined as the ratio of the present worth of tax savings to the first cost of equipment
  - Assets have inherent value but also value associated with tax benefits
  - τ is how much every dollar spent will save in taxes
    * Note that this assumes we will keep the asset around forever, so we get the tax savings for the rest of time
  - This depends on the depreciation method, tax rate, WACC, etc
  - For regular declining balance, $\tau_{db} = \dfrac{td}{i+d}$ where i is the after-tax WACC, d is the depreciation rate and t is the tax rate
    * This applies to asset disposition
  - For declining balance with half-year rule, $\tau_{1/2} = \dfrac{td}{i+d}\left(\dfrac{1+\frac{i}{2}}{1+i}\right)$
    * This applies to asset purchases
- Using tax benefit factors, the *effective first cost* is reduced because of tax savings
  - $PW(FC) = -FC + FC\tau_{1/2} = -FC(1-\tau_{1/2})$
  - The term $(1-\tau_{1/2})$ is known as the *capital tax factor* (CTF)
- The *effective salvage value* of selling an asset is reduced because of losing tax savings
  - Since the CTF assumes we keep the tax forever, when we actually sell the asset we need to correct for the tax savings that we lose
  - $PW(S) = (S - R\tau_{db})(P/F, i, N)$
  - S is the salvage value, R is the reduction in the pool due to selling the asset
  - The term $(1-\tau_{db})$ is known as the *capital salvage factor* (CSF)
- Using tax benefit factors we can calculate present worth as:
  - Use the effective first cost
  - Discount revenues by the tax rate
  - Use the effective salvage value, which depends on whether we have an open or closed book scenario
    * For open-book, $R = S$, so $PW(S) = S(1-\tau_{db})(P/F, i, N)$
    * For closed-book, $R = UCC$, and we also need to account for possible terminal loss/gain

Lecture 10, Nov 22, 2023

Inflation

Causes and Effects

- *Inflation* is a rise in the average prices of goods and services over time – the purchasing power of the dollar declines
  - The opposite would be *deflation*, which is a lot less common
- Some common causes:
  - Demand pull: people having more money (or low interest rates/taxes encouraging people to spend more)
    * This generally results from the economy doing well and is a good thing
  - Cost push: higher cost to making goods and services
  - Expanding money supply: the government printing more money, while the amount of goods and services stays the same
    * This could also result from banks lending out more money, which also creates purchasing power out of nothing
  - Expectations: inflation exists simply because people expect it to
    * Employees expect there to be inflation, so they expect their wages to rise, which in turn makes companies raise prices of goods, and so on
    * This is one of the most common causes
- Important effects of inflation:
  - Inflation benefits borrowers – we can borrow now and repay the same amount later, but the money will be worth less later
  - Inflation hurts lenders, savers, unemployed, retirees, etc
  - Menu costs (costs associated with updating prices)
  - Tax distortions (e.g. effect of tax brackets changing due to rising wages, selling assets for more than their book value because money is worth less, etc)
  - In the short run, higher inflation can lead to lower unemployment (but only in the short term)
    * Higher prices will cause businesses to hire more people, so they can make more goods to sell at the higher price
    * This happens until the business realizes that they're not making more money and this is simply a result of inflation
  - At low levels, inflation can stimulate the economy
    * The expectation of inflation pushes people to spend money now, which keeps the economy moving
  - At high levels, inflation can be disastrous
    * Hyperinflation can result in a vicious cycle, since the expectation of inflation causes people to spend money immediately, which drives up prices in turn
    * Money becomes worthless as a tool to facilitate trade, so the entire economic system collapses
- Large government banks have their primary mission as keeping inflation under control; e.g. the Bank of Canada's primary mission is to keep inflation at 2%
  - Part of the reason is some inflation can push people to spend money and stimulate the economy
  - We don't target 0% mostly because deflation is worse; if people expect prices to decline, they won't spend money, and low demand causes prices to go down and end up in a vicious cycle

Measuring Inflation – The Consumer Price Index

- Inflation is commonly measured using the Consumer Price Index (CPI)
  - This is a weighted average of the prices of some goods and services which are of primary consumer needs
  - Calculated by taking price changes for each item in a predetermined basket of goods and averaging them
- e.g. in Canada, the basket includes food, shelter, household goods, clothing, transportation, health and

personal care, recreation, alcohol and tobacco
- The CPI for a given period relates the average price of the basket to the average of the base period; current base year is 2002, and the index for the base year is always set to 100

Calculating With Inflation

- Inflation changes the value of dollars, so we define two different types of dollars to distinguish:
  - *Actual* (or *current/nominal*) dollars are expressed in the monetary units at the time the cash flows occur, i.e. this is the actual dollar amount
  - *Real* (or *constant*) dollars are expressed in the monetary units of constant purchasing power, and must always be associated with a particular date, i.e. this is how much purchasing power you actually have
- Similarly, we have two types of interest rates:
  - *Actual* interest rate (i or i_A) is the observed interest rate based on actual dollars, i.e. this is the interest rate usually given
  - *Real* interest rate (i' or i_R) is the interest rate based on constant dollars, i.e. this is the interest rate when you consider how much purchasing power you have
- Actual and real interest rates are related by $1 + i_R = \dfrac{1 + i_A}{1 + f}$, where f is the inflation rate
  - In the same time period, your money will grow by a factor of $1 + i_A$ while the price of goods grows by $1 + f$, so the growth in purchasing power is $\dfrac{1 + i_A}{1 + f}$
  - Note that for continuous compounding, this reduces to $i_R = i_A - f$
- When calculating value of cash flows, if we use actual values, then we need to adjust for inflation and use the actual MARR; if we use real values, then no need to adjust for inflation and use the real MARR
  - Note most market value interest rates are based on actual rates, e.g. loan interest rates, bond yields, CAPM, etc., except when explicitly stated
  - This is because inflation rates are hard to determine and are only published after the time period (and may even be revised), so using real rates in contracts leads to ambiguity
- Example: A company is looking to invest \$100k for a new machine that will bring benefits of \$30k for 5 years; the company expects a real return of 6% and inflation is expected to be 3%
  - If the dollar values are actual, what is the present worth of this investment?
    * Since dollar values are actual, we need to use the actual interest rate: $i_A = (1+f)(1+r_R)-1 = 9.18\%$
    * $PW = -100k + 30k(P/A, 9.18\%, 5) = 16.1k$
  - If the dollar values are real, what is the present worth of this investment?
    * Since dollar values are real, we can use the real interest rate
    * $PW = -100k + 30k(P/A, 6\%, 5) = 26.4k$
- Example: You are looking to borrow money from the bank; the bank has quoted an interest rate of 5% based on semi-annual compounding; if inflation is 2%, what is the real effective rate of interest?
  - Convert the semi-annually compounded real rate to get $r_{A,eff} = 5.063\%$
  - Therefore $r_R = \dfrac{1 + r_{A,eff}}{1 + f} - 1 = 3.002\%$
- Example: A 10 year bond with 5 years to maturity has a 4% coupon paid semi-annually; inflation is 3%; if you are targeting a real rate of 5%, how much should you pay for this bond?
  - Using the real rate we calculate the actual rate as $r_A = (1 + f)(1 + r_R) = 8.15\%$
  - Convert this to a semi-annual rate: $r_s = (1 + r_A)^{\frac{1}{2}} - 1 = 3.995\%$
  - Calculate value as $P = \dfrac{100 \cdot 4\%}{2}(P/A, r_s, 10) + 100(P/F, r_s, 10) = 83.81$
- In accounting, everything is recorded as actual dollars; depreciation is also in actual dollars so actual interest rates must be used in tax benefit factors
- Example: A machine costing \$20k has a lifetime of 7 years with a salvage value of \$6k in today's dollars; CCA rate is 20%, taxes are 25%. What is the present worth of the machine assuming a real MARR of 8% and expected inflation of 4%?
  - Convert the real MARR to an actual MARR of $r_A = (1 + f)(1 + r_R) - 1 = 12.32\%$

- The salvage value is given in today's dollars (i.e. real dollars), so we need to estimate the salvage value in 7 years as $SV_A = 6000(1+4\%)^7$
- Calculate CTF, CSF with actual MARR (not discounted by tax): $CTF = 0.8538, CSF = 0.8453$
- $PW = -20k \cdot 0.8538 + 6000(1+4\%)^7 \cdot 0.8453 \cdot (P/F, 12.32\%, 7) = -13575$
- Note we could also calculate this using real dollars; both present worth and salvage value are given in real dollars, and in the P/F factor we use the real MARR of 8%, giving the same value
  * CTF, CSF still have to be calculated with the actual MARR, but we can use either actual or real dollars as long as we discount using the right rate
  * Note that the factor of $(1+4\%)^7(P/F, 12.32\%, 7)$ is equivalent to $(P/F, 8\%, 7)$ at the real interest rate