Lecture 6, Sep 26, 2023

Example Problem: Square-Dancing Ants

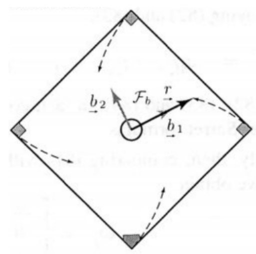

Figure 1: Example problem diagram.

- Consider 4 ants on the corners of a square with sides a; each ant directly walks towards the ant in front of it, so overall the ants all spiral inward; when the ants meet in the center, how far will each have walked?
- At any given time all the ants form a square provided their speeds are the same
- Let the speed of each ant be v, so that the path length s being walked by the ants at any given time is related to v as $v = \dot{s} = \dfrac{\mathrm{d}s}{\mathrm{d}t}$
- Construct our reference frame so that $\underset{\rightarrow}{b}_1$ and $\underset{\rightarrow}{b}_2$ point from the center of the square to two ants; $\underset{\rightarrow}{b}_3$ then points out of the page
  - This reference frame rotates since the ants move
- Let $\underset{\rightarrow}{\rho}$ be the position of one ant, so $\underset{\rightarrow}{\rho} = \underset{\rightarrow}{\mathcal{F}}_b^T \begin{bmatrix} \rho \\ 0 \\ 0 \end{bmatrix}$ since $\underset{\rightarrow}{b}_1$ directly points towards the ant
- The velocity of the ant is $\underset{\rightarrow}{v} = \underset{\rightarrow}{\mathcal{F}}_b^T \begin{bmatrix} -\frac{v}{\sqrt{2}} \\ \frac{v}{\sqrt{2}} \\ 0 \end{bmatrix} = \underset{\rightarrow}{\rho}^{\bullet} = \underset{\rightarrow}{\rho}^{\circ} + \underset{\rightarrow}{\omega}^{ba} \times \underset{\rightarrow}{\rho} = \underset{\rightarrow}{\mathcal{F}}_b^T \begin{bmatrix} \dot{\rho} \\ 0 \\ 0 \end{bmatrix} + \underset{\rightarrow}{\mathcal{F}}_b^T \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix}^{\times} \begin{bmatrix} \rho \\ 0 \\ 0 \end{bmatrix} = \underset{\rightarrow}{\mathcal{F}}_b^T \begin{bmatrix} \dot{\rho} \\ \omega\rho \\ 0 \end{bmatrix}$
  - Therefore $-\dfrac{v}{\sqrt{2}} = -\dfrac{\dot{s}}{2} = \dot{\rho}$, $\dfrac{v}{\sqrt{2}} = \dfrac{\dot{s}}{\sqrt{2}} = \omega\rho$
  - Integrating the first equation: $\rho = \dfrac{1}{\sqrt{2}}(s_0 - s)$
  - We can determine s_0 by noting that at time 0, the distance $\rho = \dfrac{a}{\sqrt{2}}$ and $s = 0$, so $s_0 = a$
  - Now we can set $\rho = 0$ to solve for s: $0 = \dfrac{1}{\sqrt{2}}(a - s) \implies s = a$
- Therefore each ant travels precisely the same length as the sides of the square