Lecture 16, Oct 31, 2023

Midterm Review

- Example: pendulum with spring and dashpot on the arm
  - Kinetic energy: $T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\dot{x}^2 + (l+x)^2\dot{\theta}^2\right)$
  - Potential energy: $V = \frac{1}{2}kx^2 - mg(l+x)\cos\theta$
  - Nonconservative forces: consider $\widehat{\delta W}_{\triangle} = Q_{x,\triangle}\delta x + Q_{\theta,\triangle}\delta\theta$
    * For a small virtual displacement δx we would have done work $f_d\delta x = -c\dot{x}\delta x$
    * A small displacement $\delta\theta$ does no non-conservative work
    * Therefore $Q_{x,\triangle} = -c\dot{x}, Q_{\theta,\triangle} = 0$
    * We could also do this using $Q_{k,\triangle} = \sum_i \underset{\rightarrow}{f}_{i,\triangle} \cdot \frac{\partial \underset{\rightarrow}{r}_i}{\partial q_k}$
  - $L = \frac{1}{2}m\left(\dot{x}^2 + (l+x)^2\dot{\theta}^2\right) - \frac{1}{2}kx^2 - mg(l+x)\cos\theta$
  - $\frac{\partial L}{\partial \dot{x}} = m\dot{x} \implies \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}$
  - $\frac{\partial L}{\partial x} = m(l+x)\dot{\theta}^2 - kx - mg\cos\theta$
  - $\frac{\partial L}{\partial \dot{\theta}} = m(l+x)^2\dot{\theta} \implies \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 2m(l+x)\dot{x}\dot{\theta} + m(l+x)^2\ddot{\theta}$
  - $\frac{\partial L}{\partial \theta} = -mg(l+x)\sin\theta$
  - $m\ddot{x} - m(l+x)\dot{\theta}^2 + kx - mg\cos\theta = -c\dot{x}$
  - $m(l+x)^2\ddot{\theta} + 2m(l+x)\dot{x}\dot{\theta} + mg(l+x)\sin\theta = 0$