Lecture 11, Oct 12, 2023

Orbital Motion and Gravity

- Kepler's laws:
  1. The orbits of planets are ellipses with the sun at one focus
  2. An orbiting planet sweeps out an equal area in equal time, i.e. $\frac{\mathrm{d}A}{\mathrm{d}t} = c$
  3. The square of the orbital period of a planet is proportional to the cube of the mean distance from the sun, i.e. $T^2 \propto a^3$
- Kepler tells us the shape of the orbits without need for epicycles, but still does not tell us the fundamental force that causes such motion
- Newton's law of gravitation: $\underset{\rightarrow}{f}{}^a_b = \frac{G m_a m_b}{r_3} \underset{\rightarrow}{r}{}^a_b$ where $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$ is the universal gravitation constant
  1. The force of gravity is central
  2. The force of gravity varies according to the inverse-square law
  3. The force of gravity is universal
- Kepler's second law can be derived directly from the fact that gravity is a central force
- Kepler's third law implies that this central force varies according to the inverse square law
- The universality of gravity can be determined by comparing the acceleration of an object on earth to the acceleration of the moon, after taking into account the inverse square law
- Inside a hollow shell, there is no gravitational force due to the inverse square law
- Inside a solid sphere, the gravitational force varies proportional to r (since the mass varies as r^3 and the gravitational force varies as r^{-2})
  - This means if we dug a hole through the earth and dropped a particle, it would exhibit simple harmonic motion, since the force is proportional to distance from the core