Lecture 4, Jan 17, 2022

Time-Independent Schrödinger's Equation

- Assume $\Psi(x,t) = \phi(t)\psi(x)$, giving $i\hbar\frac{1}{\phi}\frac{\mathrm{d}\phi}{\mathrm{d}t} = -\frac{\hbar^2}{2m\psi}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V$
  - Since the two sides depend on different variables we argue that both need to be equal to a constant A
  - Therefore $i\frac{A}{\hbar}\phi = \frac{\mathrm{d}\phi}{\mathrm{d}t}$ and $A = -\frac{\hbar^2}{2m\psi}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V$
- The first equation has a general solution of $\phi = Ce^{-i\frac{A}{\hbar}t}$ where C is a constant
  - We conclude that A must have units of energy, so we let $A = E$
  - The oscillation frequency is going to depend on the energy; more energy means the wavefunction oscillates faster in time
- Substituting $A = E$ into the second equation and multiplying by ψ we get $-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V\psi = E\psi$, which is known as the time-independent Schrödinger equation