Lecture 2, Jan 13, 2022

Wave-Particle Duality and The Bohr Model

- Classically particles and waves are distinct, but in the 20th century we found matter can behave like waves and waves can behave like particles
  - The photoelectric effect shows that light (which was known to be electromagnetic waves) behaving like particles through the photoelectric effect, described by Planck and Einstein
  - Exciting hydrogen atoms and other gases only emits specific discrete spectral lines
  - Shooting electrons through a diffraction grating (Ni crystals) show diffraction behaviour which is wavelike
- De Broglie relation: $p = \dfrac{h}{\lambda} = \hbar k$ proposes that particles can exhibit wavelike behaviour, relating classical momentum, which is a property of particles, to a wavelength (from $E^2 = p^2c^2 + m^2c^4$ and $E = h\nu = \dfrac{hc}{\lambda}$)
- Rutherford-Bohr model: negatively charged electron orbiting a positively charged electron
  - If we equate the centripetal force on the electron $\dfrac{mv^2}{r}$ to the Coulomb force $\dfrac{e^2}{4\pi\varepsilon_0 r^2}$ we get $v^2 = \dfrac{e^2}{4\pi\varepsilon_0 mr}$
  - Adding energy, i.e. adding velocity, will decrease the radius; however we get a continuous relation between r and v while in reality this is not true
  - Bohr added interference through $n\lambda = I_{orbit} = 2\pi r$ with n as a quantisation constant, because the radius has to be an integer multiple of the wavelength for the waves to constructively interfere
  - Invoking de Broglie's relation $mv = \dfrac{\hbar}{\lambda}$ and equating λ we get $v = \dfrac{hn}{2\pi mr}$ and angular velocity $L = mvr = n\hbar$, i.e. angular momentum is quantized in units of $\hbar$
  - Equating the expressions for v and solving for r, the allowed radii are $r_n = \dfrac{\hbar^2\varepsilon_0 n^2}{\pi me^2} = a_{Bohr}n^2$ where n is a positive integer, and the *Bohr radius* is the smallest radius, about $5.3 \times 10^{-11}\,\mathrm{m}$
  - Solving for the total energy (sum of kinetic and Coulomb energy) gives energy inversely proportional to n^2 (and negative because the larger the n the less energy it has)
  - Using this we can write an expression for the energy emitted as the electron jumps between states, and we get the Rydberg equation back!
- Even though the Bohr model explains the hydrogen spectrum, it fails:
  - Assumes circular electron orbits, which is fundamentally incorrect, so it only works for hydrogen and fails miserably for helium
  - It treats electrons as things with definite radius and momentum, which is in violation of the uncertainty principle
  - Does explain where the emission lines are, but not why some are brighter than the others