Lecture 15, Feb 14, 2022

Non Steady State Diffusion

- Fick's Second law: $\dfrac{\partial C}{\partial t} = D\dfrac{\partial^2 C}{\partial x^2}$
  - Boundary conditions: at $t = 0$, $C = C_0$ for $x \leq x \leq \infty$ (assume bar has preexisting concentration of copper)
  - At $t > 0$, $C = C_s$ for $x = 0$ (constant surface concentration), and $C = C_0$ for $x = \infty$
- Solution is $\dfrac{C(x,t) - C_0}{C_s - C_0} = 1 - \operatorname{erf}\left(\dfrac{x}{2\sqrt{Dt}}\right)$ where the error function $\operatorname{erf}(z) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^z e^{-y^2}\,\mathrm{d}y$
  - With increasing time, the concentration profile flattens and the material moves deeper into the bar
  - The $\sqrt{Dt}$ is the characteristic length that the diffusing atom has moved in time t
- Example: An FCC Iron-Carbon alloy initially containing 0.20% carbon by weight is carburized at an elevated temperature and in an atmosphere that gives a constant surface carbon concentration of 1.0% by weight. After 49.5h the concentration of carbon is 0.35% by weight at a position 4.0mm below the surface. Determine the temperature at which the treatment was carried out.
  - Using the solution equation we can get $\operatorname{erf}(z) = 0.8125$
  - Go to the table to find the value of z using an interpolation, and then solve for $D = 2.6 \times 10^{-11}\ \mathrm{m}^2/\mathrm{s}$
  - Now we have D, we need D_0 and Q to find the temperature
- Diffusion is faster for:
  - Open crystal structures
  - Smaller diffusing atoms
  - Lower density materials

Phase Diagrams

- Given the composition of a material and the temperature, we can determine how many phases it forms, what the composition each phase is and what the amount of each phase is
- Solubility limit: Maximum concentration for which only a single phase solution exists (i.e. a homogeneous solution e.g. syrup)
- Phase diagrams have 2 axes: composition (x) and temperature (y)
  - The composition axis goes between no substance 1, all substance 2, to all substance 1, no substance 2 (wt%)
  - The diagram is divided into two big regions; on the left we have a homogeneous solution (below the solubility limit); on the right we have the dual phase region where both solid and solution exist
- Components: Elements or compounds which are present in the alloy (e.g. Al or Cu)
- Phases: Physically and chemically distinct material regions that form (e.g. α, β)
  - e.g. aluminum-copper alloy under a microscope has β phases (lighter) and α phases (darker)
- State variables: temperature and composition
  - Higher temperature leads to greater solubility and moves us from a dual phase region to a single phase region
  - Changing the composition also changes between phase regions (isothermal changes)
  - Pressure is also a state variable but not in this course
- Solubility is affected by Hume-Rothery rules
  - Not meeting the criteria gives typically limited solubility
- e.g. nickel and copper are soluble in one another for all proportions
  - In the phase diagram there's a thin transition area where both liquid and solid solution exist, as the crystals have to from from the solid first
- A binary isomorphous system (e.g. copper and nickel) has only 2 components, and complete solubility of one component in another
  - i.e. it is possible to have the same phase for 0 to 100% composition
  - The *liquidus* divides the single phase liquid region and the mixed region; the *solidus* divides the single phase solid region and the mixed region