Lecture 35, Apr 11, 2022

Example Problems

- Consider the sequence: $\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k, \boldsymbol{x}_k \in {}^n\mathbb{R}$, and let $\boldsymbol{A} \in {}^n\mathbb{R}^n$ be diagonalizable with real eigenvalues; show that if all $|\lambda_\alpha| < 1$, then $\boldsymbol{x}_k \to 0$ as $k \to \infty$
  - Note $\boldsymbol{x}_k = \boldsymbol{A}^k\boldsymbol{x}_0$
  - $\lim\limits_{k\to\infty} \boldsymbol{x}_k = \lim\limits_{k\to\infty} \boldsymbol{A}^k\boldsymbol{x}_0 = \lim\limits_{k\to\infty} \boldsymbol{P}\boldsymbol{\Lambda}^k\boldsymbol{P}^{-1}\boldsymbol{x}_0$
  - $\lim\limits_{k\to\infty} \boldsymbol{\Lambda}^k = \lim\limits_{k\to\infty} \begin{bmatrix} \lambda_1^k & & \\ & \lambda_2^k & \\ & & \ddots \end{bmatrix} = \mathbf{0}$
  - Therefore $\lim\limits_{k\to\infty} \boldsymbol{P}\boldsymbol{\Lambda}^k\boldsymbol{P}^{-1}\boldsymbol{x}_0 = \mathbf{0}$