Lecture 15, Feb 15, 2022

Rank

- Definition: The *rank* of $\boldsymbol{A}$, denoted $\operatorname{rank}\boldsymbol{A}$, is the common dimension of its row and column space: $\operatorname{rank}\boldsymbol{A} \equiv \dim\operatorname{row}\boldsymbol{A} = \dim\operatorname{col}\boldsymbol{A}$
  - Can also be expressed in different ways, e.g. number of nonzero rows in the RREF, the number of leading ones in the RREF, etc
- Properties of rank:
  - Property I: $\operatorname{rank}\boldsymbol{A} = \operatorname{rank}\tilde{\boldsymbol{A}}$
  - Property II: $\operatorname{rank}\boldsymbol{A} = \operatorname{rank}\boldsymbol{A}^T$
  - Property III: $\operatorname{rank}\boldsymbol{U}\boldsymbol{A} \leq \operatorname{rank}\boldsymbol{A}$
    * $\operatorname{row}\boldsymbol{U}\boldsymbol{A} \subseteq \operatorname{row}\boldsymbol{A}$ by Prop. I
    * $\operatorname{rank}\boldsymbol{U}\boldsymbol{A} = \operatorname{rank}\boldsymbol{A}$ when $\boldsymbol{U}$ is invertible since $\operatorname{row}\boldsymbol{U}\boldsymbol{A} = \operatorname{row}\boldsymbol{A}$ by Prop. I
    * Similarly $\operatorname{rank}\boldsymbol{A}\boldsymbol{V} \leq \operatorname{rank}\boldsymbol{A}$ and $\operatorname{rank}\boldsymbol{A}\boldsymbol{V} = \operatorname{rank}\boldsymbol{A}$ if (but not only if) $\boldsymbol{V}$ is invertible