Lecture 1, Jan 11, 2022

Review: Matrices

- $A = [a_{ij}] \in {}^m\mathbb{R}^n$ is a general $m \times n$ matrix in the reals
  - ${}^m\mathbb{R}$ is the set of all $m \times 1$ real matrices (i.e. columns)
  - $\mathbb{R}^n$ is the set of all $1 \times n$ real matrices (i.e. rows)
  - We can also have matrices of $\mathbb{C}, \mathbb{Z}, \mathbb{Q}$, etc
- Matrix multiplication: $A \in {}^m\mathbb{R}^n, B \in {}^n\mathbb{R}^p \implies C = AB = [c_{ij}] \in {}^m\mathbb{R}^p, c_{ij} = \displaystyle\sum_{k=1}^{n} a_{ik}b_{kj}$
- Transpose swaps rows and columns: $A \in {}^m\mathbb{R}^n \implies A^T \in {}^n\mathbb{R}^m$
- Trace is the sum of the main diagonal $A \in {}^n\mathbb{R}^n \implies \operatorname{tr} A = \displaystyle\sum_{i=1}^{n} a_{ii}$ (for square matrices only)
- Determinant $\det A$ for square matrices
- Inverse A^{-1} also for square matrices; exists only when $\det A \neq 0$ (i.e. matrix is full rank); $AA^{-1} = A^{-1}A = I$
  - Pseudoinverses exist for nonsquare matrices
  - For square matrices $AB = I \implies BA = I$
- $(A^T)^{-1} = (A^{-1})^T$ so sometimes we denote $(A^{-1})^T = A^{-T}$
- Symmetric matrices $A^T = A$, skew symmetric (or anti-symmetric) $A^T = -A$
- Identity denoted as a boldface 1
- Matrix addition is associative and commutative; matrix multiplication is associative and not commutative
- We have closure under matrix addition and scalar multiplication; that is after adding two matrices and multiplying by a scalar we still get a matrix in the same dimensions