Lecture 21, Mar 7, 2022

Vectors, Lines and Planes

- $\vec{a} = (x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$
- In general vectors don't have a particular location in space where it starts (they represent displacement instead of a location)
- Radius vectors denoted $\vec{r}$ start at $(0, 0, 0)$
- A plane is described by $ax + by + cz = d$ or $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
  - (a, b, c) is the normal vector of the plane, (x_0, y_0, z_0) is a point on the plane
- Planes are defined uniquely by a point on the plane and a normal vector
  - Let $\vec{n} = (n_1, n_2, n_3)$ be the normal vector and $\vec{r}_0 = (x_0, y_0, z_0)$ be the point on the plane
  - Let $p = (x, y, z)$ be on the plane connected to the origin by $\vec{r}$
  - We constrain the plane by noting that any $\vec{r} - \vec{r}_0$ is in the plane and thus normal to $\vec{n}$, therefore $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$
    * $n(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$
- A line is described by $\vec{r} = \vec{r_0} + t\vec{v} \implies (x, y, z) = (x_0, y_0, z_0) + t(v_1, v_2, v_3)$ parametrically or $\dfrac{x - x_0}{v_1} = \dfrac{y - y_0}{v_2} = \dfrac{z - z_0}{v_3}$ (the 2-equation symmetric form)
- Lines are defined uniquely by a point on the line and a parallel vector
  - Let $\vec{v}$ be the direction vector and $\vec{r}_0$ be the point on the line
  - Any other point $\vec{r}$ on the line must have $\vec{r} - \vec{r}_0 \propto \vec{v} \implies \vec{r} = \vec{r}_0 + t\vec{v}$
- Parametric equations in 3D are often written as position vectors $\vec{r}(t)$