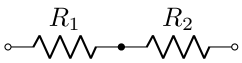
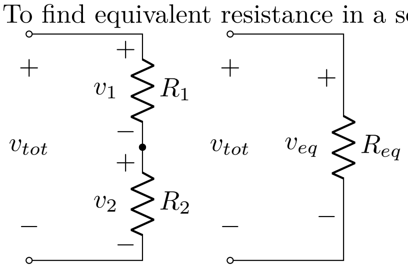
Lecture 9, Feb 2, 2022

Simplifying Series and Parallel Resistors

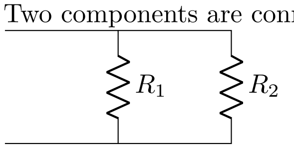
- Two components are connected *in series* if they're connected back-to-back, and at the point of connection there is no other current path:



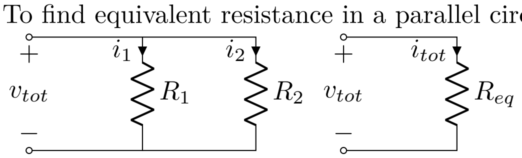
- To find equivalent resistance in a series circuit, compare:



  - KVL gives: $-v_{tot} + v_1 + v_2 = 0$
    * Applying Ohm's law: $v_1 = R_1 i_{tot}$ and $v_2 = R_2 i_{tot}$
    * Substituting the voltages back in: $-v_{tot} + R_1 i_{tot} + R_2 i_{tot} = 0 \implies v_{tot} = (R_1 + R_2) i_{tot}$
    * Compare this to Ohm's law for the second circuit, we see that the equivalent resistance is $R_1 + R_2$
    * This generalizes to any number of resistors to give $R_{eq} = R_1 + R_2 + \cdots + R_n$
    * In the extreme cases, if one resistor is an open circuit $R = \infty$, the entire circuit can be considered as an open connection; if one resistor is a short circuit, then it wouldn't have any effect
- Two components are connected *in parallel* if they share two common nodes:



- To find equivalent resistance in a parallel circuit, compare:



  - KCL gives: $i_{tot} = i_1 + i_2$
    * KVL gives: $v_{tot} = R_i i_1 = R_2 i_2 \implies i_1 = \frac{v_{tot}}{R_1}, i_2 = \frac{v_{tot}}{R_2}$
    * $i_{tot} = \frac{v_{tot}}{R_1} + \frac{v_{tot}}{R_2}$
    * Compare this to circuit 2 we get $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \implies R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$
      - Alternatively, the equivalent conductance of two resistors in parallel is the sum of the conductances
    * The conductance relation generalizes to any number of resistors; however $\frac{R_1 R_2}{R_1 + R_2}$ becomes $\frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$ for 3 resistors, $\frac{R_1 R_2 R_3 R_4}{R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4}$ and so on
    * In the extreme cases, if one resistor is a short circuit, then the entire circuit can be considered a short circuit; if one resistor is an open connection, then it does not have an impact (since $\frac{1}{R} \to 0$)