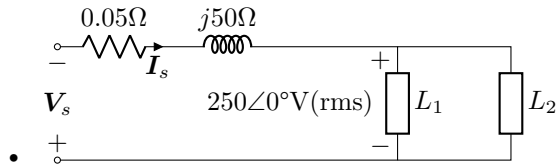# Lecture 39, Apr 12, 2022

AC Power Example

- In the circuit above, L_1 absorbs 8kW, $PF = 0.8$ leading; L_2 absorbs 20kVA, $PF = 0.6$ lagging; find the power factor of the combined load, $\boldsymbol{I}_s$, and the average power loss of the transmission line
    * L_1's power draw has units of watts, which indicates that it's an average power; L_2's power draw is in volt-amps, indicating either apparent, instantaneous or complex power, but since it's a real time-independent expression it's an apparently power
        - Complex power can only be entirely real when the power factor is 1, since in that case the angle is zero
    * Note the complex power of two combined loads is simply the sum, so for part 1, we want to find the complex power of both loads, add them and then find the angle to get the PF
    * First we need the apparent power of L_1 since we only have the average power:
        - $P_1 = V_{rms}I_{rms}PF \implies V_{rms_1}I_{rms_1} = \dfrac{8\text{kW}}{0.8} = 10\text{kW} = S_1$
        - Now we can find angle by $\theta = \cos^{-1}(0.8) = \pm 36.87°$
        - Since the power factor is leading, the angle is negative, so $\boldsymbol{S}_1 = 10000\angle -36.87°\text{VA}$
        - Now we can calculate $\boldsymbol{S}_1 = P + jQ = 8000 + j10000\sin(-36.8°) = 8000 - j6000\text{VA}$
    * Do the same for L_2:
        - $\theta = \cos^{-1}(0.6) = \pm 53.13°$, since the power factor is lagging, this is positive
        - $P_2 = S_2 \cdot 0.6 = 12\text{kW}$
        - Combined $\boldsymbol{S}_2 = 12000 + j16000\text{VA}$
    * Adding the two loads gives a combined complex power of $20000 + j10000\text{VA}$, which has an angle of 26.56°, giving it a power factor of 0.894 lagging
    * Recall $\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* \implies \boldsymbol{I}_s^* = \dfrac{\boldsymbol{S}}{\boldsymbol{V}} \implies \boldsymbol{I}_s = \dfrac{\boldsymbol{S}^*}{\boldsymbol{V}^*} = \dfrac{22361\angle -26.56°}{250\angle 0°} = 89.44\angle -26°\text{A}$
    * To find the average power loss, we find the active power of the resistor and inductor (since the inductor has only reactive power we can ignore it)
        - $P_{loss} = R|I_s|^2 = 0.05\Omega \cdot (89.44\text{A})^2 = 400\text{W}$

Power Factor Correction

- For most practical loads, the power factors are lagging since most loads can be modelled by $\boldsymbol{Z} = R + j\omega L$; some of the power is loss through the transmission line, which is proportional to the square of the magnitude of the current phasor
- The more the voltage and current are out of phase, the more inefficient the power transmission is (large magnitude of line current which leads to high loss, but low power factor for the actual load)
- By properly choosing a capacitance in parallel with the load, we can cancel or reduce the imaginary part of $\boldsymbol{I}_{load}$, putting the voltage and current more in phase and increasing transmission efficiency ($\theta_v - \theta_i$ becomes smaller, power factor becomes bigger, which is why this method is called *power factor correction*)
- If the corrected power factor is 1, we call it *full power factor correction*; otherwise it's a *partial power factor correction*
- Example: For the previous circuit, we had a frequency of 60Hz
    - We want to choose $\boldsymbol{S}_C$ such that $\boldsymbol{S} + \boldsymbol{S}_C$ is entirely real so it has a power factor of 1
    - This means for the capacitor $Q = -j10000$, but also for a capacitor $Q = jX_C|\boldsymbol{I}_C|^2 = jX_C\left|\dfrac{\boldsymbol{V}}{jX_C}\right|^2 =$

$$j\frac{|\boldsymbol{V}|^2}{X_C}$$

- Therefore $\dfrac{|\boldsymbol{V}^2|}{X_C} = -10000 \implies X_C = -6.25\Omega$

- From this reactance we can find the capacitance as $X_C = -\dfrac{1}{2\pi fC} \implies C = 424.4\mu\text{F}$