Lecture 38, Apr 11, 2022

Complex Power

- So far all of the types of power are real numbers; complex power is the only type of power that is complex
- The *complex power* for a sinusoidal AC circuit is defined as $\boldsymbol{S} = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^* = \boldsymbol{V}_{rms}\boldsymbol{I}^*_{rms}$ where $\boldsymbol{V} = V_m\angle\theta_v, \boldsymbol{I} = I_m\angle\theta_i$
  - Complex power also has units of volt-amps
- We can also write complex power as $\boldsymbol{S} = \frac{1}{2}(V_m\angle\theta_v)(I_m\angle-\theta_i) = \frac{1}{2}V_mI_m\angle(\theta_v - \theta_i)$
  - Recall the apparent power $S = \frac{1}{2}V_mI_m$, so the complex power has an amplitude equal to the apparent power
- Using Euler's formula, in rectangular form $\boldsymbol{S} = V_{rms}I_{rms}\cos(\theta_v - \theta_i) + jV_{rms}I_{rms}\sin(\theta_v - \theta_i)$
- Complex power is related to average/active and reactive power by $\boldsymbol{S} = P_{ave} + jQ$
- The power factor is the cosine of the angle of the complex power
- Consider an impedance $\boldsymbol{Z} = R + jX$, complex power is
$$\begin{aligned}\boldsymbol{S} &= \boldsymbol{V}_{rms}\boldsymbol{I}^*_{rms}\\ &= \boldsymbol{Z}\boldsymbol{I}_{rms}\boldsymbol{I}^*_{rms}\\ &= (R + jX)|\boldsymbol{I}_{rms}|^2\\ &= RI^2_{rms} + jXI^2_{rms}\\ &= P_{ave} + Q\end{aligned}$$
- $\boldsymbol{S}, P$ and Q form a triangle in the complex plane
- Conservation of power: the algebraic sum of all the complex powers in a circuit is zero, $\sum_{k=1}^{n}\boldsymbol{S}_k = 0$ where n is the number of circuit elements
  - Since complex power has a real and imaginary component, conservation of power holds for both average and reactive power separately