Lecture 37, Apr 11, 2022

Power In Terms of RMS Values

- Average power can be expressed in terms of the RMS voltage and current as $P_{ave} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$
  - Using phasors $P_{ave} = \mathrm{Re}(\boldsymbol{V}_{rms} \boldsymbol{I}^*_{rms})$
- Likewise for reactive power $Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$
  - Using phasors $Q = \mathrm{Im}(\boldsymbol{V}_{rms} \boldsymbol{I}^*_{rms})$
- For an impedance, recall for $\boldsymbol{Z} = R + jX$, average power $P_{ave} = \mathrm{Re}\left(\frac{(R + jX)|\boldsymbol{I}|^2}{2}\right) = \frac{1}{2} R I_m^2 = R I_{rms}^2$
- For reactive power, $Q = \mathrm{Im}\left(\frac{(R + jX)|\boldsymbol{I}|^2}{2}\right) = \frac{1}{2} X I_m^2 = X I_{rms}^2$

Apparent Power & Power Factor

- Active power $P_{ave} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$, can be divided into two terms, the *apparent power* $S = \frac{1}{2} V_m I_m = V_{rms} I_{rms}$ (in volt-amps), and the *power factor* $PF = \cos(\theta_v - \theta_i)$
- Consider an impedance $\boldsymbol{Z}$ with voltage $\boldsymbol{V}$ and current $\boldsymbol{I}$, $\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{V_m}{I_m} \angle(\theta_v - \theta_i)$
  - Notice the power factor is the cosine of the angle of the impedance here $PF = \cos(\angle \boldsymbol{Z})$
- The power factor does not uniquely determine the impedance, since cosine is an even function, so given only a power factor we don't know if the angle of the impedance is positive or negative
  - Define a power factor as *leading* if current leads voltage, i.e. $\theta_v - \theta_i < 0$, and *lagging* if current lags voltage, i.e. $\theta_v - \theta_i > 0$
- For a resistor, $\angle \boldsymbol{Z} = 0$ so the power factor is $\cos(0) = 1$, neither lagging nor leading
  - This is the only type of impedance to have a power factor of 1
- For an inductor, $\angle \boldsymbol{Z} = 90°$ so the power factor is $\cos(90°) = 0$, and lagging since $\angle \boldsymbol{Z} > 0$
- For a capacitor, $\angle \boldsymbol{Z} = -90°$ so the power factor is also 0, but this time leading
- For an RL circuit, $\boldsymbol{Z} = R + j\omega L$ so $0° < \angle \boldsymbol{Z} < 90°$ so the power factor is between 0 and 1 lagging
- For an RC circuit, $\boldsymbol{Z} = R - \frac{j}{\omega C}$ and the power factor is between 0 and 1 leading
- For an RLC circuit, $\boldsymbol{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$ so the power factor is between 0 and 1, either leading or lagging, depending on the relative values of the inductance and capacitance