Lecture 36, Apr 8, 2022

Root-Mean-Square (RMS) Power

- The RMS (aka *effective value*) of a periodic signal $x(t)$ with period T is given by $x_{rms} = x_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)\,\mathrm{d}t}$
- AC voltages are often expressed in RMS
- Consider $v(t) = V_m \cos(\omega t + \theta_v)$, $T = \frac{2\pi}{\omega}$
  - $v_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)\,\mathrm{d}t}$
    $= \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \theta_v)\,\mathrm{d}t}$
    $= \sqrt{\frac{V_m^2}{T}\int_0^T \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\theta_v)\,\mathrm{d}t}$
    $= \sqrt{\frac{V_m^2}{T} \cdot \frac{1}{2}T}$
    $= \sqrt{\frac{V_m^2}{2}}$
    $= \frac{V_m}{\sqrt{2}}$
    * Note the second term in the integral cancels since we're integrating a sinusoidal function over a multiple of its period
- Similarly for currents $i_{rms} = \frac{I_m}{\sqrt{2}}$
- North American 110V AC outlets have an amplitude of $110\sqrt{2}$V
- RMS values are associated with the energy in the signal and applies to non-sinusoidal periodic waveforms as well