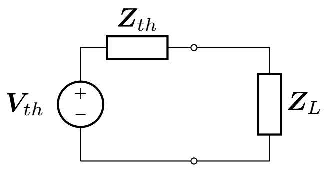
Lecture 35, Apr 6, 2022

Maximum Average Power

- Like maximum power transfer in DC circuits, we would like to know the impedance we should connect to an arbitrary linear AC circuit to maximize average power consumed by the impedance
- Find the Thevenin equivalent, and form the circuit:



  - $\boldsymbol{V} = \boldsymbol{Z}_L \boldsymbol{I}$
- Active power for the impedance is $P_L = \mathrm{Re}\left(\frac{\boldsymbol{V}\boldsymbol{I}^*}{2}\right)$

$$= \mathrm{Re}\left(\frac{(R_L + jX_L)|\boldsymbol{I}|^2}{2}\right)$$

$$= \frac{1}{2}R_L|\boldsymbol{I}|^2$$

  - In terms of the circuit parameters, $\boldsymbol{I} = \frac{\boldsymbol{V}_{th}}{\boldsymbol{Z}_{th} + \boldsymbol{Z}_L} = \frac{\boldsymbol{V}_{th}}{(R_{th} + R_L) + j(X_{th} + X_L)}$
  - $|\boldsymbol{I}|^2 = \frac{|\boldsymbol{V}_{th}|^2}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}$
  - So $P_L(R_L, X_L) = \frac{1}{2}R_L\frac{|\boldsymbol{V}_{th}|^2}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}$
  - Now we have 2 parameters to optimize, R_L and X_L
  - $\frac{\partial P_L}{\partial X_L} = R_L\frac{0 - 2(X_{th} + X_L)|\boldsymbol{V}_{th}|^2}{4((R_{th} + R_L)^2 + (X_{th} + X_L)^2)^2} = 0$
    * $\boldsymbol{V}_{th}$ cannot be zero, which means the only possibility is to have $X_{th} = -X_L$
  - $\frac{\partial P_L}{\partial R_L} = \frac{|\boldsymbol{V}_{th}|^2((R_{th} + R_L)^2 + (X_{th} + X_L)^2) - 2(R_{th} + R_L)|\boldsymbol{V}_{th}|^2 R_L}{4((R_{th} + R_L)^2 + (X_{th} + X_L)^2)^2} = 0$
    * Again $\boldsymbol{V}_{th}$ can't be zero so we can cancel it out
    * $((R_{th} + R_L)^2 + (X_{th} + X_L)^2) - 2(R_{th} + R_L)R_L = 0$
    * Factor out $R_L + R_{th}$: $(R_{th} + R_L)(R_{th} + R_L - 2R_L) + (X_{th} + X_L) = 0$

$$\implies R_{th}^2 - R_L^2 + (X_{th} + X_L)^2 = 0$$

$$\implies R_L = \sqrt{R_{th}^2 + (X_{th} + X_L)^2}$$

    * Note we already derived above that $X_L = -X_{th}$, therefore $R_L = R_{th}$
- Without constraints, the impedance that maximizes power transfer is $\boldsymbol{Z}_L = R_{th} - X_{th}$
  - Max power is simply $\frac{1}{2}R_L\frac{|\boldsymbol{V}_{th}|^2}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2} = 8\frac{|\boldsymbol{V}_{th}|^2}{R_{th}}$