Lecture 34, Apr 4, 2022

Different Types of Power in AC Circuits

- Instantaneous power: $P(t) = \frac{V_m I_m}{2}\cos(\theta) + \frac{V_m I_m}{2}\cos(2\omega t - \theta)$
- *Average power* (aka *real* or *active power*): $P_{ave} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$
  - If we integrate the instantaneous power over one period, the time-variant second term cancels out, and we're left with only the first term
  - Average power is in watts
- Using phasors, if $\begin{cases} v(t) = V_m \cos(\omega t) \\ i(t) = I_m \cos(\omega t - \theta) \end{cases} \implies \begin{cases} \boldsymbol{V} = V_m \angle 0° \\ \boldsymbol{I} = I_m \angle -\theta \end{cases}$, then $\frac{\boldsymbol{V}\boldsymbol{I}^*}{2} = \frac{(V_m \angle 0°)(I_m \angle \theta)}{2} = \frac{V_m I_m}{2}\angle\theta = \frac{V_m I_m}{2}\cos(\theta) + j\frac{\boldsymbol{V}_m \boldsymbol{I}_m}{2}\sin(\theta)$, so $P_{ave} = \mathrm{Re}\left(\frac{\boldsymbol{V}\boldsymbol{I}^*}{2}\right)$
- $P(t) = \frac{V_m I_m}{2}\cos(\theta) + \frac{V_m I_m}{2}(2\omega t - \theta)$

  $= \frac{V_m I_m}{2}\cos(\theta) + \frac{V_m I_m}{2}\cos(2\omega t)\cos(\theta) + \frac{V_m I_m}{2}\sin(2\omega t)\sin(\theta)$

  $= \frac{V_m I_m}{2}\cos(\theta)\left(1 + \cos(2\omega t)\right) + \frac{V_m I_m}{2}\sin(\theta)\sin(2\omega t)$
- *Reactive power*: $Q = \frac{V_m I_m}{2}\sin(\theta)$
  - Reactive power has units of volt-amp-reactive, more commonly known as VAR
- Note that from the relation derived above for average power, we have $Q = \mathrm{Im}\left(\frac{\boldsymbol{V}_m \boldsymbol{I}_m^*}{2}\right)$
- Instantaneous power in terms of average and reactive power: $P(t) = P_{ave}(1 + \cos(2\omega t)) + Q\sin(2\omega t)$
- Conservation holds for both active and reactive power

AC Power for R, L, and C

- For a resistor voltage and current are in phase, so $\theta_v = \theta_i \implies \theta = 0 \implies P_{ave} = \frac{V_m I_m}{2}, Q = 0$
  - For instantaneous power, we are left with only the first term: $P(t) = \frac{V_m I_m}{2}(1 + \cos(2\omega t))$
  - Plotting this gives a sinusoid with a DC offset equal to the amplitude, i.e. the value varies between 0 and $V_m I_m$
  - $P_{ave} = \mathrm{Re}\left(\frac{\boldsymbol{V}\boldsymbol{I}^*}{2}\right) = \mathrm{Re}\left(\frac{R|I|^2}{2}\right) = \mathrm{Re}\left(\frac{RI_m^2}{2}\right) = \frac{1}{2}RI_m^2$
    * This is exactly like the expression for power for DC
  - $Q = \mathrm{Im}\left(\frac{\boldsymbol{V}\boldsymbol{I}^*}{2}\right) = 0$
- For an inductor $\theta_v - \theta_i = 90°$ so we're only left with $P(t) = \frac{V_m I_m}{2}\sin(2\omega t)$
  - This is a sinusoid with no DC offset
    * Every half-period, an inductor absorbs energy (positive power), and the next half-period it releases the same amount of energy back
  - $P_{ave} = 0$
  - $Q = \frac{V_m I_m}{2}\sin(90°) = \frac{V_m I_m}{2}$
  - Alternatively $Q = \mathrm{Im}\left(\frac{j\omega L\boldsymbol{I}\boldsymbol{I}^*}{2}\right) = \frac{1}{2}\omega L I_m^2 = \frac{1}{2}X_L I_m^2$ where X_L is the reactance
- For a capacitor $\theta = -90°$ so $P(t) = -\frac{V_m I_m}{2}\sin(2\omega t)$
  - This is the same as an inductor but negated; every half-period it absorbs energy, and then in the next half-cycle it releases power back
  - $P_{ave} = 0$

- $Q = \frac{V_m I_m}{2} \sin(-90°) = -\frac{V_m I_m}{2}$
- Alternatively $Q = \text{Im}\left(\frac{-j\boldsymbol{I}\boldsymbol{I}^*}{\omega C}\right) = -\frac{1}{2\omega C} I_m^2 = \frac{1}{2} X_C I_m^2$ where X_C is the reactance