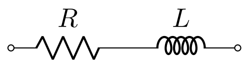
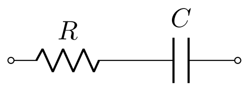
Lecture 31, Mar 30, 2022

Impedance of RL, RC, LC, and RLC circuits

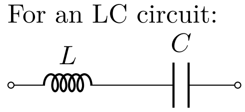
- For an RL circuit:



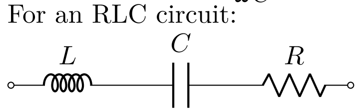
  - $\boldsymbol{Z}_{RL} = \boldsymbol{Z}_R + \boldsymbol{Z}_L = R + j\omega L$
    * The real part of the impedance for an RL circuit is the resistance of the resistor; the imaginary part is ωL, frequency times the inductance of the inductor
    * Combining R and L gives both a resistance and a reactance
    * The angle depends on both R and L; if $\boldsymbol{Z}_R \gg \boldsymbol{Z}_L$ then $\angle \boldsymbol{Z}_{RL} \to 0$; if $\boldsymbol{Z}_R \ll \boldsymbol{Z}_L$ then $\angle \boldsymbol{Z}_{RL} \to 90°$
    * The phase difference is $0 < \theta_v - \theta_i < 90°$; greater resistance leads to less phase difference, while greater inductance leads to more phase difference
    * Voltage leads current by some amount between 0° and 90°
- For an RC circuit:



  - $\boldsymbol{Z}_{RC} = \boldsymbol{Z}_R + \boldsymbol{Z}_C = R - \dfrac{j}{\omega C}$
    * This time the angle is between 0° and 90° since the imaginary part (reactance) is negative
    * $\boldsymbol{Z}_R \gg \boldsymbol{Z}_C \implies \angle \boldsymbol{Z}_{RC} \to 0$ and $\boldsymbol{Z}_R \ll \boldsymbol{Z}_C \implies \angle \boldsymbol{Z}_{RC} \to -90°$
    * Voltage lags current by some amount between 0° and 90° (current leads voltage)
- For an LC circuit:



  - $\boldsymbol{Z}_{LC} = \boldsymbol{Z}_L + \boldsymbol{Z}_C = j\left(\omega L - \dfrac{1}{\omega C}\right)$
    * The impedance of an LC circuit is entirely imaginary (no resistance)
    * The imaginary part can be positive or negative, depending on the relative values of the inductance and capacitance
      - $\omega L > \dfrac{1}{\omega C} \implies \text{Im}\, \boldsymbol{Z}_{LC} > 0$, and voltage leads current by 90°
      - $\omega L < \dfrac{1}{\omega C} \implies \text{Im}\, \boldsymbol{Z}_{LC} < 0$, and voltage lags current by 90°
- For an RLC circuit:

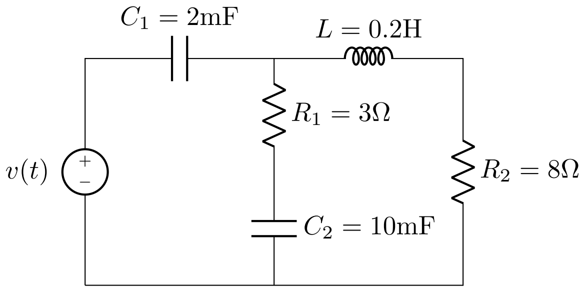


  - $\boldsymbol{Z}_{RLC} = \boldsymbol{Z}_R + \boldsymbol{Z}_C + \boldsymbol{Z}_L = R + j\left(\omega L - \dfrac{1}{\omega C}\right)$
    * The real part is positive, the imaginary part can be positive or negative depending on the relative values of inductance and capacitance
    * The angle is between −90° and 90°; sign follows the same pattern as for an LC circuit

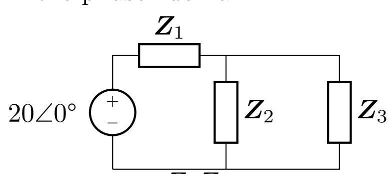
Sinusoidal Steady State Analysis

- Since all the laws and techniques (KVL, KCL, etc) still hold in the phasor domain, we can analyze AC circuits in the same way
- Convert the circuit into phasor domain (resistances, inductances, and capacitances to impedances), and use $\boldsymbol{Z} = \dfrac{\boldsymbol{V}}{\boldsymbol{I}}$ in the same way that $R = \dfrac{v}{i}$ is used in DC circuits
  - Convert the phasors back to time domain afterwards if desired
- The only difference is that complex phasors are used instead of real numbers

- Example:



  - $v(t) = 20\cos(50t)V$, find $i(t)$
    * Convert the voltage to a phasor: $\boldsymbol{V} = 20\angle 0°$
    * For the 2mF capacitor, $\boldsymbol{Z}_1 = \dfrac{-j}{\omega C_1} = \dfrac{-j}{50 \cdot 2 \times 10^{-3}} = -j10\Omega$
    * For the series RC connection in the middle, $\boldsymbol{Z}_2 = R_1 - \dfrac{j}{\omega C_2} = 3 - \dfrac{j}{50 \cdot 10 \times 10^{-3}} = 3 - j2\Omega$
    * For the series RL connection, $\boldsymbol{Z}_3 = R_2 + j\omega L = 8 + j50 \cdot 0.2 = 8 + j10\Omega$
    * In the phasor domain:



    * $\boldsymbol{Z}_{eq} = \boldsymbol{Z}_1 + \dfrac{\boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_2 + \boldsymbol{Z}_3} = 3.22 - j11.07\Omega = 11.53\angle - 73.5°$
    * $\boldsymbol{I} = \dfrac{\boldsymbol{V}}{\boldsymbol{Z}} = \dfrac{20\angle 0°}{11.53\angle - 73.5°} = 1.73\angle 73.5°\text{A}$
    * Back to the time domain, $i(t) = 1.73\cos(50t + 73.5°)$