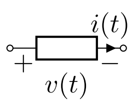
Lecture 30, Mar 28, 2022

KVL and KCL in the Phasor Domain

- Regular KVL on a loop gives
$$\sum_{i=1}^{n} v_i(t) = 0$$
$$\implies \sum_{i=1}^{n} V_i \cos(\omega t + \alpha_i) = 0$$
$$\implies \text{Re}\left\{\sum_{i=1}^{n} V_i e^{j\alpha_i} e^{j\omega t}\right\} = 0$$
$$\implies \text{Re}\left\{\left(\sum_{i=1}^{n} V_i e^{j\alpha_i}\right) e^{j\omega t}\right\} = 0$$
$$\implies \sum_{i=1}^{n} \boldsymbol{V}_i = 0$$
- Similarly for KCL on a node, $\sum_{i=1}^{n} \boldsymbol{I}_n = 0$
- Both KVL and KCL hold in the phasor/frequency domain

Impedance

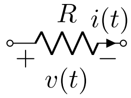
- Consider a general circuit element:



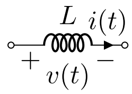
  - Suppose $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$
- *Impedance* for this element is defined as $\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \left(\frac{V_m}{I_m}\right) \angle(\theta_v - \theta_i)$ where $\boldsymbol{V}$ and $\boldsymbol{I}$ are the phasors for the voltage and current (assuming PSC)
  - Impedance is similar to resistance in a DC circuit
  - Unit for impedance is ohms
  - Expressed in rectangular form, $\boldsymbol{Z} = R + jX$
    * The real part R is the *resistance*
    * The imaginary part X is the *reactance*
    * Both have units of ohms
- Similar to conductance in DC circuits, for AC define the *admittance* as $\boldsymbol{Y} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = \frac{1}{\boldsymbol{Z}}$
  - Admittance has the same units as conductance, Siemens or mhos
  - Expressed in rectangular form, $\boldsymbol{Y} = G + jB$, where G is the *conductance*, and B is the *susceptance*, both having units of Siemens

Impedance Relations for Passive Components

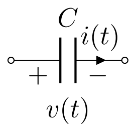
- For a resistor:



  - $\boldsymbol{Z}_R = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{R\boldsymbol{I}}{\boldsymbol{I}} = R$
    * The impedance of a resistor is just the resistance of that resistor
    * A resistor has no reactance; voltage and current are always in phase
- For an inductor:



  - $\boldsymbol{Z}_L = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{j\omega L\boldsymbol{I}}{\boldsymbol{I}} = j\omega L$
    * The impedance of an inductor is entirely imaginary, i.e. it has no resistance but ωL reactance
    * Note the impedance is dependent on frequency; when $\omega \to 0$, $\boldsymbol{Z}_L \to 0$, and when $\omega \to \infty$, $\boldsymbol{Z}_L \to \infty$
    * An inductor affects higher frequency signals more
    * Intuitively, decreasing the frequency makes the current closer to DC conditions, under which the inductor is a short circuit
- For a capacitor:



  - $\boldsymbol{Z}_C = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{\boldsymbol{V}}{j\omega C\boldsymbol{V}} = \frac{1}{j\omega C} = -\frac{j}{\omega C}$
    * The impedance of a capacitor is also pure imaginary; resistance is zero and reactance is $-\frac{1}{\omega C}$
    * When $\omega \to 0$, $\boldsymbol{Z}_C \to \infty$; when $\omega \to \infty$, $\boldsymbol{Z}_C \to 0$
    * A capacitor affects lower signal frequencies more
    * Intuitively, lower frequencies are closer to DC conditions, under which a capacitor is an open circuit

Equivalent Impedances

- Consider a number of impedance elements in series; all elements have the same $\boldsymbol{I}$ so across each there is a voltage of $\boldsymbol{V}_i = \boldsymbol{Z}_i\boldsymbol{I}$, so $\boldsymbol{V} = \boldsymbol{I}\sum_{i=1}^{n}\boldsymbol{Z}_i \implies \frac{\boldsymbol{V}}{\boldsymbol{I}} = \sum_{i=1}^{n}\boldsymbol{Z}_i$
- The equivalent impedance for a series connection is $\boldsymbol{Z}_{eq} = \sum_{i=1}^{n}\boldsymbol{Z}_i$
- For a parallel connection the impedance is $\frac{1}{\boldsymbol{Z}_{eq}} = \sum_{i=1}^{n}\frac{1}{\boldsymbol{Z}_i}$