Lecture 29, Mar 25, 2022

Phasors (Continued)

- Phasors are often shown on a phasor diagram on a complex plane, where the length is the magnitude, and the angle is the phase offset
- Alternatively they can be expressed as $\boldsymbol{V} = V_m \angle \alpha = V_m e^{j\alpha}$
- Example: Phasor for $v(t) = V_m \cos(377t + 60°)$ is $\boldsymbol{V} = V_m e^{j60°}$; phasor for $i(t) = I_m \sin(377t + 30°)$ is $\boldsymbol{I} = I_m e^{-j60°}$
  - Note phasors are defined in terms of cosines, which is why 30° becomes −60°
  - Also note how the frequencies don't affect the phasors
- Phasors can be used to add together two sinusoids in frequency domain
- Example: Using phasors, find the sum of $i_1(t) = 4\cos(\omega t + 30°)$ and $i_2(t) = 5\sin(\omega t - 20°)$
  - $\boldsymbol{I}_1 = 4e^{j30°}, \boldsymbol{I}_2 = 5e^{-j110°}$
  - Convert phasors to rectangular format to add them: $\boldsymbol{I}_1 = 4\cos(30°) + 4i\sin(30°), \boldsymbol{I}_2 = 5\cos(-110°) + 5i\sin(-110°)$
  - $\boldsymbol{I}_1 + \boldsymbol{I}_2 = 1.754 - 2.698j = 3.218e^{-j56.98°}$

Derivatives and Integrals of Sinusoids

- $v(t) = V_m \cos(\omega t + \alpha) \implies \boldsymbol{V} = V_m e^{j\alpha}$
- $$\frac{\mathrm{d}v}{\mathrm{d}t} = -V_m \omega \sin(\omega t + \alpha) = V_m \omega \cos\left(\omega t + \alpha + \frac{\pi}{2}\right)$$
  $\implies \boldsymbol{V} = V_m \omega e^{j\left(\alpha + \frac{\pi}{2}\right)} = \left(\omega e^{j\frac{\pi}{2}}\right)\left(V_m e^{j\alpha}\right) = j\omega V_m e^{j\alpha}$
- Taking the time-domain derivative multiplies the phasor by $j\omega$
- $$\int v(t)\,\mathrm{d}t = \frac{V_m}{\omega}\sin(\omega t + \alpha) = \frac{V_m}{\omega}\cos\left(\omega t + \alpha - \frac{\pi}{2}\right) = \frac{1}{j\omega} V_m e^{j\alpha}$$
  $\implies \boldsymbol{V} = \frac{V_m}{\omega} e^{j\left(\alpha - \frac{\pi}{2}\right)} = \left(\frac{1}{\omega} e^{-j\frac{\pi}{2}}\right)\left(V_m e^{j\alpha}\right) = -\frac{j}{\omega} V_m e^{j\alpha}$
- Taking the time-domain integral multiplies the phasor by $\frac{1}{j\omega}$ (i.e. divide by $j\omega$)

Phasor Relations for R, L, C

- Suppose $i(t) = I_m \cos(\omega t + \theta_i) \implies \boldsymbol{I} = I_m e^{j\theta_i}$ passes through a resistor
  - Assuming PSC, by Ohm's law, $v(t) = Ri(t) = RI_m \cos(\omega t + \theta_i) \implies \boldsymbol{V} = RI_m e^{j\theta_i} \implies \boldsymbol{V} = R\boldsymbol{I}$
- $\boldsymbol{V} = R\boldsymbol{I}$ for a resistor; voltage and current are in phase
  - In the phasor diagram, a resistor only changes the length of the phasor and does not rotate it
- For an inductor: $v(t) = L\frac{\mathrm{d}i}{\mathrm{d}t}$ in the time domain, so in the phasor domain, $\boldsymbol{V} = j\omega L\boldsymbol{I}$
  - Since $j = e^{j90°}$, an inductor introduces a 90° phase difference (voltage leads current by 90°)
  - The magnitude is scaled by ωL
  - In the phasor diagram, the two phasors have an angle of 90° relative to each other
- For a capacitor: $i(t) = C\frac{\mathrm{d}v}{\mathrm{d}t}$ in the time domain, so in the phasor domain, $\boldsymbol{I} = j\omega C\boldsymbol{V}$ or $\boldsymbol{V} = \frac{1}{j\omega C}\boldsymbol{I}$
  - A capacitor also introduces a 90° phase difference (voltage lags current by 90°)