Lecture 28, Mar 23, 2022

Sinusoids and Phasors

- Many AC sources generate sinusoidal voltages and currents (e.g. power generators, the power grid)
- Generally $v(t) = V_m \sin(\omega t + \alpha)$ for a sinusoidal voltage
  - V_m is the amplitude of the sinusoidal voltage in volts
  - ω is the angular frequency, in rad/s
  - t is time in seconds, so that ωt has units of rad
    - The period is related by $T_0 = \dfrac{2\pi}{\omega}$, or $\omega = \dfrac{2\pi}{T_0}$
    - T_0 is the *fundamental period*, the smallest possible period
  - α is the phase/phase angle/initial phase/etc in radians
    - Note sometimes α may be given in degrees, in which case you need to multiply by $\dfrac{\pi}{180^\circ}$ to convert to radians
    - A signal with larger α *leads* another signal, while a smaller α *lags* another signal
    - Phase leads or lags are commonly expressed with an angle in the $-\pi$ to π range (e.g. "leading by 270°" is unconventional)
  - We also define *frequency* (as opposed to angular frequency) as $f = \dfrac{1}{T_0} = \dfrac{\omega}{2\pi}$ with units of $\mathrm{s}^{-1} = \mathrm{Hz}$
- Note the phase difference between two sinusoidal signals is $\alpha_1 - \alpha_2$ in radians, but to convert this to time we need to divide by ω
  - If two ω are different for two signals, the phase difference is undefined
  - Cosine has an additional phase offset of $+\dfrac{\pi}{2}$ when compared to sine
  - Adding a phase offset of 180° negates the sign
  - $\sin(\alpha \pm 180^\circ) = -\sin(\alpha)$
  - $\cos(\alpha \pm 180^\circ) = -\cos(\alpha)$
  - $\sin(\alpha \pm 90^\circ) = \pm\cos(\alpha)$
  - $\cos(\alpha \pm 90^\circ) = \mp\sin(\alpha)$
- Often sinusoidal signals are defined with a cosine function as $v(t) = V_m \cos(\omega t + \alpha)$
- Using Euler's formula, $V_m e^{j(\omega t + \alpha)} = V_m \cos(\omega t + \alpha) + jV_m \sin(\omega t + \alpha)$ (where $j^2 = -1$)
  - $v(t) = \mathrm{Re}\left(V_m e^{j(\omega t + \alpha)}\right) = \mathrm{Re}\left(V_m e^{j\alpha} e^{j\omega t}\right)$
  - For a given ω, $v(t)$ is uniquely determined by a complex number $V_m e^{j\alpha}$ (magnitude is V_m, argument is α)
    - This complex number is the *phasor* for the voltage, indicated by $\boldsymbol{V} = V_m e^{j\alpha}$
- Similarly for currents, the phasor for $i(t) = I_m \cos(\omega t + \alpha)$ is $\boldsymbol{I} = I_m e^{j\alpha}$