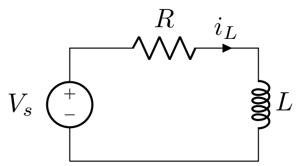
Lecture 27, Mar 21, 2022

Step Response of an RL Circuit

- Consider an RL circuit with an voltage source that turns on at $t = 0$, with the inductor initially charged with a current of I_0:



  - KVL:
$$\begin{aligned} & -V_s + Ri_L(t) + L\frac{\mathrm{d}i_L}{\mathrm{d}t} = 0 \\ \implies & i_L(t) - \frac{V_s}{R} = -\frac{L}{R}\frac{\mathrm{d}i_L}{\mathrm{d}t} \\ \implies & \int \frac{1}{i_L(t) - \frac{V_s}{R}}\,\mathrm{d}i_L = \int -\frac{1}{L/R}\,\mathrm{d}t \\ \implies & \ln\left(i_L(t) - \frac{V_s}{R}\right) + K = -\frac{t}{L/R} \\ \implies & i_L(t) = \frac{V_s}{R} + Ae^{-\frac{t}{L/R}} \end{aligned}$$
    - Using the initial condition of $i_L(0) = I_0 \implies A = I_0 - \dfrac{V_s}{R}$
    - $i_L(t) = \dfrac{V_s}{R} + \left(I_0 - \dfrac{V_s}{R}\right)e^{-\frac{t}{L/R}} = \dfrac{V_s}{R} + \left(I_0 - \dfrac{V_s}{R}\right)e^{-\frac{t}{\tau}}$
    - $v_L(t) = L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} = (V_s - RI_0)\,e^{-\frac{t}{L/R}}$
- The current starts at I_0 and exponentially decays towards $\dfrac{V_s}{R}$, the final value for current (at which point the inductor is a short circuit)
  - At $t = 0$ this is accompanied by a jump in the voltage
- The time constant τ is given by $\dfrac{L}{R}$
- As with the RC case, $i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-\frac{t}{\tau}}$
  - $i_L(\infty) = \dfrac{V_s}{R}$ because the inductor becomes a short circuit
  - $i_L(0) = I_0$