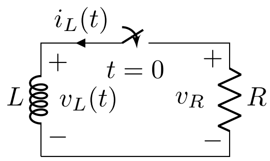
Lecture 26, Mar 18, 2022

Source-Free RL Circuits

- Consider a source-free RL circuit:



  - Initial condition: $i_L(0) = I_0$
    * Since the current of an inductor cannot change abruptly, we find the current
    * KVL: $v_L - v_R = 0 \implies L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} + iR = 0$
    * Solving the differential equation: $\displaystyle\int \frac{1}{i_L}\,\mathrm{d}i_L = -\int \frac{1}{\frac{L}{R}}\,\mathrm{d}t \implies \ln(i_L(t)) + K = -\frac{t}{\frac{L}{R}}$
    * Solution is $i_L(t) = Ae^{-\frac{t}{L/R}}$
    * Using the initial condition gives $A = I_0$, giving $i_L(t) = I_0e^{-\frac{t}{L/R}} = I_0e^{-\frac{t}{\tau}}$ where $\tau = \dfrac{L}{R}$ is the time constant for an RL circuit
      - Larger time constant means slower decay
      - Similar to τ for a capacitor, the time constant can be found by the intersection of the tangent line at $t = 0$ with the time axis
    * Voltage: $v_L = L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} = -RI_0e^{-\frac{t}{\tau}}$