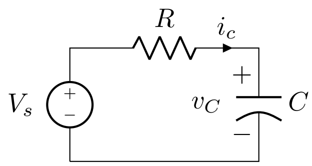
Lecture 25, Mar 16, 2022

Step Response of an RC Circuit

- What happen if we add an independent source to the RC circuit?
- 

  - KVL gives: $-V_s + Ri_C + v_C = 0$
    * $i_C = C\frac{\mathrm{d}v_C}{\mathrm{d}t} \implies -V_s + RC\frac{\mathrm{d}v_C}{\mathrm{d}t} + v_C(t) = 0 \implies \frac{\mathrm{d}v_C}{\mathrm{d}t} = \frac{-(v_c(t) - V_s)}{RC}$
    * This is again separable: $\int \frac{1}{v_C(t) - V_s}\,\underset{.}{\mathrm{v}}_C = -\int \frac{1}{RC}\,\mathrm{d}t$

$$\implies \ln(v_C(t) - V_s) = -\frac{t}{RC} + K$$

$$\implies v_C(t) - V_s = Ae^{-\frac{t}{RC}}$$

$$\implies v_C(t) = Ae^{-\frac{t}{RC}} + V_s$$

    * Using the initial condition that $v_C(0^+) = V_0 \implies V_0 = A + V_s \implies A = V_0 - V_s$
    * Finally, $v_C(t) = V_s + (V_0 - V_s)e^{-\frac{t}{RC}}$
- As $t \to \infty$ we have $v_C(t) \to V_s$
  - At $t = 0$ there is a sharp corner as the voltage starts either increasing or decreasing and exponentially decaying to V_s
- To find the current: $i_C(t) = C\frac{\mathrm{d}v_C}{\mathrm{d}t} = \frac{V_s - V_0}{R}e^{-\frac{t}{RC}}$
  - At $t = 0$ there is a discontinuity in the current as it jumps to $\frac{V_s - V_0}{R}$; as $t \to \infty, i_C \to 0$
- In general $v_C(t) = v_C(\infty) + (v_C(0) - v_C(\infty))e^{-\frac{t}{RC}}$
- This can also be applied to any general linear circuit connected to the capacitor by finding its Thevenin equivalent
- The result can be broken down into two parts: a contribution from the initial voltage, and a part from the source: $v_C(t) = V_s(1 - e^{-\frac{t}{RC}}) + V_0e^{-\frac{t}{RC}}$
  - The second part is exactly the behaviour of the source-free circuit
  - The first part is called the *forced response*, and the second part is the *natural response*