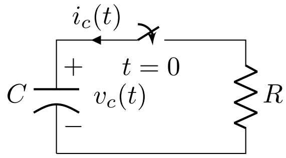
# Lecture 24, Mar 14, 2022

## Source-Free RC Circuits

- In a first-order transient circuit, the relationship between current and voltage can be described by a first-order differential equation
  - These are either RC or RL circuits (resistors and capacitors/inductors)
  - They can have sources or no sources
- Consider a source-free RC circuit:




  - Suppose before time 0 the capacitor is energized to $v_c(0^-) = V_0$, and then at time 0, the switch is closed and the energizing circuit is removed
    * KVL gives: $v_c(t) + Ri_c(t) = 0 \implies v_c(t) + RC\dfrac{\mathrm{d}v_c}{\mathrm{d}t} = 0 \implies v_c = -RC\dfrac{\mathrm{d}v_c}{\mathrm{d}t}$
    * This is a separable equation: $\displaystyle\int \frac{1}{v_c}\,\mathrm{d}v_c = \int -\frac{1}{RC}\,\mathrm{d}t \implies \ln(v_c) + K = -\frac{t}{RC}$
    * Rearranging: $v_c(t) = Ae^{-\frac{t}{RC}}$
    * Solving for initial conditions with $v_c(0) = V_0$, we obtain $v_c(t) = V_0 e^{-\frac{t}{RC}}$
      - Note we can do this because a capacitor's voltage cannot change abruptly, so $v_c(0^+) = v_c(0^-) = V_0$
      - We could not have started with current because we don't know the current at $0^+$
    * The current is then $i_c(t) = C\dfrac{\mathrm{d}v_c}{\mathrm{d}t} = -\dfrac{V_0 C}{RC}e^{-\frac{t}{RC}} = -\dfrac{V_0}{R}e^{-\frac{t}{RC}}$ or $\dfrac{V_0}{R}e^{-\frac{t}{RC}}$ not following PSC
      - Note there is a discontinuity at time 0 as the current starts at 0 and jumps to $\dfrac{V_0}{R}$, and then decays to 0
- In a source-free RC circuit the voltage across the capacitor follows an exponential decay to 0
  - Large $RC$ causes slower decay; small $RC$ causes faster decay
  - Let $\tau = RC$ be the *time constant* of the RC circuit; $\tau$ characterizes how fast the voltage decays
  - $\tau$ can be found by finding the tangent at $t = 0$, and finding where the tangent intersects the horizontal axis
    * $\dfrac{\mathrm{d}v_c}{\mathrm{d}t}(0) = -\dfrac{V_0}{RC}$ so tangent is $y = V_0 - \dfrac{V_0 t}{RC}$; therefore when $t = \tau = RC$ the tangent intersects the time axis
  - $\tau$ has the same unit as time (seconds), so that the argument of the exponential is unitless