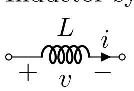
Lecture 23, Mar 11, 2022

Series and Parallel Connections of Capacitors

- Suppose we have n capacitors $c_1, \cdots, c_n$ connected in parallel
  - Each capacitor has a current $i_k = c_k \frac{\mathrm{d}v_k}{\mathrm{d}t}$ but the capacitors all have the same voltage, so $i_k = c_k \frac{\mathrm{d}v}{\mathrm{d}t}$
  - To find the equivalent capacitance, we need to find the total current
  - $i_{tot} = \sum_{k=1}^{n} i_k = \frac{\mathrm{d}v}{\mathrm{d}t} \sum_{k=1}^{n} c_k \implies c_{eq} = \sum_{k=1}^{n} c_k$
- The equivalent capacitance for capacitors in parallel is the sum of all the capacitances
- Suppose we have n capacitors $c_1, \cdots, c_n$ connected in series
  - All capacitors have the same current and each has a voltage v_k
  - KVL gives $v_{tot} = \sum_{k=1}^{n} v_k \implies \frac{\mathrm{d}v_{tot}}{\mathrm{d}t} = \sum_{k=1}^{n} \frac{\mathrm{d}v_k}{\mathrm{d}t} = \sum_{k=1}^{n} \frac{1}{c_k} i = i \sum_{k=1}^{n} \frac{1}{c_k} \implies \frac{1}{c_{eq}} = \sum_{k=1}^{n} \frac{1}{c_k}$
- The equivalent capacitance for capacitors in series is the reciprocal of the sum of the reciprocals of the capacitances
  - For c_1 and c_2 in series, $c_{eq} = \frac{c_1 c_2}{c_1 + c_2}$
- The behaviour in series vs parallel for capacitors is opposite that of resistors

Inductors

- An inductor consists of a coil of conducting wire with a core of any material
- Like a capacitor, an inductor stores energy, this time in a magnetic field generated as current passes through it
  - Since the energy density for a magnetic field is much larger than that of an electric field, the energy that can be stored in an inductor is much larger than a capacitor
- Inductor symbol:



- For an inductor, voltage is related to current by $v = L\frac{\mathrm{d}i}{\mathrm{d}t}$; an inductor is the dual of a capacitor
  - L is the *inductance* and has units of henry H
    - The larger the value of L, the more energy can be stored in the inductor
    - L depends on the kind of core used in the inductor
  - This relation only holds if PSC holds
- To get current from voltage we can integrate: $i(t_2) = i(t_1) + \frac{1}{L}\int_{t_1}^{t_2} v \,\mathrm{d}t$ or $i(t) = i(0) + \frac{1}{L}\int_0^t v(\tau)\,\mathrm{d}\tau$
- Properties of inductors:
  1. If the current is constant, then the voltage is always 0
     - In a DC circuit (in steady state) the inductor can be modelled by a short
  2. The current of an inductor cannot change abruptly since that would create an infinite voltage
- Energy of a capacitor:
$$\begin{aligned} W(t_2) - W(t_1) &= \int_{t_1}^{t_2} P(t)\,\mathrm{d}t \\ &= \int_{t_1}^{t_2} v(t)i(t)\,\mathrm{d}t \\ &= \int_{t_1}^{t_2} Li(t)\frac{\mathrm{d}i}{\mathrm{d}t}\,\mathrm{d}t \\ &= L\int_{t_1}^{t_2} i\,\mathrm{d}i \\ &= \frac{1}{2}L(i^2(t_2) - i^2(t_1)) \end{aligned}$$

  - Assuming no magnetic field at $t = 0$, $W(t) = \frac{1}{2}Li^2(t)$
  - Like an ideal capacitor, an ideal inductor does not dissipate energy and only stores it
- The equivalent inductance of inductors in series is the sum of the inductances; in parallel it's the reciprocal of the sum of the reciprocals (like resistors)