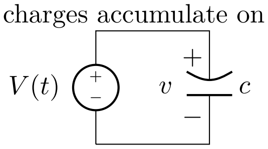
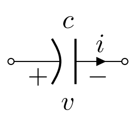
Lecture 22, Mar 9, 2022

Capacitors

- A capacitor consists of 2 conducting plates separated by an insulator; when connected to a voltage, charges accumulate on the plates, creating an electric field and storing energy:



  - The accumulated charge is proportional to the voltage: $q(t) = cv(t)$
    * c is the *capacitance*, and is determined by the physical characteristics of the capacitor (similar to resistance for a resistor)
    * Capacitance has units of Coulombs per volt or farads: $\mathrm{C/V} = \mathrm{F}$
      - In practice one farad is a very large capacitance; most capacitors are in the order of microfarads or smaller
- Translating this into current: $i = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}cv(t) \implies i(t) = c\frac{\mathrm{d}v}{\mathrm{d}t}$
  - Current passing through a capacitor is proportional to the rate of change of voltage
  - This relation holds if PSC holds; otherwise $i = -c\frac{\mathrm{d}v}{\mathrm{d}t}$
- In the other direction: $\int_{t_1}^{t_2} c\frac{\mathrm{d}v}{\mathrm{d}t}\,\mathrm{d}t = \int_{t_1}^{t_2} i(t)\,\mathrm{d}t \implies v(t_2) - v(t_1) = \frac{1}{c}\int_{t_1}^{t_2} i(t)\,\mathrm{d}t$
  - $v(t) = v(0) + \frac{1}{c}\int_0^t i(\tau)\,\mathrm{d}\tau$
  - To find the voltage of a capacitor at time t, integrate the current
  - We need both the current function and a known value of $v(t)$, unlike with current from voltage where we only need the voltage function
- Properties of capacitors:



  1. If the voltage is constant (i.e. DC), then current is always 0, since $\frac{\mathrm{d}v}{\mathrm{d}t}$ is 0
     - A capacitor can be modelled as an open circuit in a DC circuit
  2. The voltage of a capacitor cannot change abruptly; a discontinuity in voltage creates an infinite $\frac{\mathrm{d}v}{\mathrm{d}t}$ and infinite current
- Find energy of a capacitor:
$$\begin{aligned} W(t_2) - W(t_1) &= \int_{t_1}^{t_2} P(t)\,\mathrm{d}t \\ &= \int_{t_1}^{t_2} v(t)i(t)\,\mathrm{d}t \\ &= \int_{t_1}^{t_2} cv(t)\frac{\mathrm{d}v}{\mathrm{d}t}\,\mathrm{d}t \\ &= c\int_{t_1}^{t_2} v\,\mathrm{d}v \\ &= \frac{1}{2}c(v^2(t_2) - v^2(t_1)) \end{aligned}$$
  - Assuming capacitor is unchanged at $t = 0$ (i.e. $v(0) = 0$), $W(t) = \frac{1}{2}cv^2(t)$
  - An ideal capacitor does not dissipate energy; it only stores and delivers energy
- Although an ideal capacitor stops all DC current, a physical capacitor has some leakage current
- A real capacitor can be modelled as an ideal capacitor in parallel with a *leakage resistance* of R_L, typically in the hundreds of megaohms