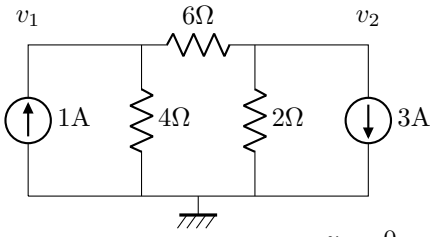
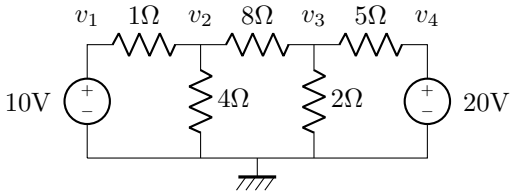
Lecture 12, Feb 7, 2022

Nodal Analysis

- Nodal analysis is an algorithmic method for circuit analysis; it finds the node voltages at every node in the circuit
  - Define *node voltage* as the voltage between a node and a reference point (common ground), with positive polarity at the node and negative polarity at the reference point
  - The reference (ground) node is typically denoted with a ground symbol: ⏚ or ⏚
- Apply KCL for every node in terms of the node voltages
  - Voltage between two nodes is the difference of their nodal voltages
  - $v_{AB} = v_A - v_B$ and $v_{BA} = v_B - v_A$
- Procedure:
  1. Find all the nodes in the circuit and label them, choose one as ground
     - Choice of ground node is arbitrary but sometimes it can simplify the math
     - Choose the node that's connected to the highest number of voltage sources; prefer independent sources over dependent sources
  2. Assume current directions/signs (negative for current entering node, positive for current leaving)
  3. Write KCL for all the ungrounded notes
     - Current sources: we have voltage directly, resistors: use Ohm's law
     - If we have a voltage source between the ground node and another node, we can get the voltage of that node directly
     - Usually we always write the current that leaves a node via a resistor since it gives a positive sign
  4. Solve the system for the nodal voltages and use the nodal voltages to find anything else needed
- Example circuit 1:



  - KCL at node 1: $-1 + \dfrac{v_1 - 0}{4} + \dfrac{v_1 - v_2}{6} = 0$
    - KCL at node 2: $3 + \dfrac{v_2 - 0}{2} + \dfrac{v_2 - v_1}{6} = 0$
    - Solution: $v_1 = \dfrac{2}{3}\text{V}, v_2 = -\dfrac{13}{2}\text{V}$
- Example circuit 2:



  - We get $v_1 = 10\text{V}, v_2 = 20\text{V}$ immediately
    - Node 2: $\dfrac{v_2 - v_1}{1} + \dfrac{v_2 - 0}{4} + \dfrac{v_2 - v_3}{8} = 0$
    - Node 3: $\dfrac{v_3 - v_2}{8} + \dfrac{v_3 - 0}{2} + \dfrac{v_3 - v_4}{5} = 0$
    - Solution: $v_2 = 7.82\text{V}, v_3 = 6.03\text{V}$