Lecture 32 (2-16), Dec 1, 2022

Relativistic Doppler Shift

- For classical waves the observed frequency is $f = \frac{f_0}{1 - \frac{v_{src}}{v_{wave}}}$ for a source moving towards an observer
- For a relativistic frequency shift, we need to take time dilation into account
  - $f = \frac{1}{\gamma}\frac{f_0}{1 - \frac{v_s}{v_w}} = f_0\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} = f_0\sqrt{\frac{1+\beta}{1-\beta}} = f_0\sqrt{\frac{c+v}{c-v}}$
  - For a source moving away, flip the signs
- Example: Redshift of stars suggests that they're moving away

Relativistic Momentum

- Consider a simple collision of two particles with mass m, colliding head-on with speed u towards each other; after the collision both particles change to the y direction
  - $\vec{u}_{1,i} = (u, 0, 0)$
  - $\vec{u}_{2,i} = (-u, 0, 0)$
  - Total momentum $\vec{p}_{tot} = 0$
  - After the collision the first particle becomes $\vec{u}_{1,f} = (0, u, 0)$ and the second particle becomes $\vec{u}_{2,f} = (0, -u, 0)$ so that $\vec{p}_{tot} = 0$ is conserved
- Now consider frame s' moving with velocity $v = u$ (i.e. a frame moving with particle 1 pre-collision)
  - Before the collision $\vec{u}'_{2i}$ is the only particle moving
  - The velocity of this particle would be $u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-u-u}{1 - \frac{(-u)u}{c^2}} = \frac{-2u}{1+\frac{u^2}{c^2}}$
  - $\vec{p}' = \frac{-2mu}{1+\frac{u^2}{c^2}}\hat{x}$
  - After the collision, both particles have both an x and y component
  - $u'_{xf} = \frac{u_{xf} - v}{1 - \frac{u_{xf}v}{c^2}} = \frac{0-u}{1-0} = -u$
  - $u'_{yf} = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} = \frac{u}{\gamma}$
  - $\vec{p}' = m(-u)\hat{x} + m\frac{u}{\gamma}\hat{y} + m(-u)\hat{x} + m\frac{-u}{\gamma}\hat{y} = -2mu\hat{x}$
  - Momentum is seemingly not conserved, which violates our first postulates that all laws of physics stay the same
- Therefore we need to use a new definition of momentum
- Define the relativistic momentum $\vec{p} = \gamma_p m\vec{u}$ where $\gamma_p = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$
  - With this definition our momentum is now conserved