Lecture 30 (2-14), Nov 28, 2022

Special Relativity Definitions

- Define two dimensionless quantities, β and γ
  - $\beta = \dfrac{v}{c}$
  - $\gamma = \dfrac{1}{1-\beta^2} = \dfrac{1}{1-\frac{v^2}{c^2}}$
  - γ is close to 1 when velocity is not close to c; when v approaches c γ grows very quickly and diverges
    * When $\gamma \approx 1$ the problem is non-relativistic
  - When $\beta \ll 1$ we can use the Taylor expansion and write $\gamma = (1-\beta^2)^{-\frac{1}{2}} \approx 1 + \dfrac{\beta^2}{2}$
- Usually time is plotted on the vertical axis and space on the horizontal axis
  - If we plot out the trajectory of light in both directions we get two lines
  - Define a *light cone* as the region between the lines of the trajectory of light
    * Events in the negative light cone can influence events at the origin
    * Events at the origin can influence events in the positive light cone
  - When we take this to multiple dimensions, the “light triangle” becomes a light cone as we introduce other dimensions
- We scale the axes of the spacetime diagram so that they have the same units, so that the light cone lines have slope 1
- On a spacetime diagram within a light cone we can draw *world lines*, the trajectory of an object in spacetime
  - Nothing (starting at the origin) can be outside the light cone since that means it’d be going faster than the speed of light

The Light Clock and Time Dilation

- Consider reference frames s and s' moving horizontally with velocity v; an experiment is happening inside s' where an observer reflects light between two mirrors a height of h apart
- In s', $\Delta t' = \dfrac{2h}{c}$
- In s, light has to travel diagonally since s' is moving horizontally relative to s
  - When light is reflected from the first mirror, s' has moved by an amount $\dfrac{v(\Delta t)}{2}$; when the light is detected again s' moves another amount
    * Since light has to travel a longer distance, but the speed of light is the same, so time must increase!
  - $\left(\dfrac{c\Delta t}{2}\right)^2 = \left(\dfrac{v\Delta t}{2}\right)^2 + h^2 \implies \Delta t^2(c^2 - v^2) = 4h^2 \implies \Delta t = \dfrac{2h}{\sqrt{c^2 - v^2}} = \dfrac{2h}{c\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{2h}{c}\gamma = \gamma\Delta t'$
  - $\gamma > 1$ so $\Delta t' > \Delta t$, i.e. a time interval measured in s' is shorter than a time interval measured in s
  - Time has been “dilated” for the observer outside s'
- The observer in s' records the *proper time*; everyone else records a dilated time
  - $\Delta t'$ is the proper time since it’s the time measured at the same place as the event, often written as Δt_0
- $\Delta t = \gamma \Delta t_0$
- We see evidence of this in muon decay