Lecture 28 (2-12), Nov 22, 2022

Potential Barrier

- If the potential step goes to 0 again we have a potential barrier
- $V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 \leq x \leq a \\ 0 & x > a \end{cases}$
- In the third region we have $\psi_{III} = Fe^{ikx}$ (note there is no e^{-ikx} term since there will not be a wave moving to the left in this region)
- Inside the barrier $C = 0$ is no longer true because we can normalize it even with $C \neq 0$
- Calculate $R = \left|\frac{B}{A}\right|^2, T = \left|\frac{F}{A}\right|^2$
- After matching boundary conditions, $T = \frac{16E(V_0 - E)}{V_0^2}e^{-2\alpha a}$, assuming $V_0 \gg E$, where $\alpha^2 = \frac{2m(V_0 - E)}{\hbar^2}$
  - The expression for T and R in general are quite complicated, but if we take the limit $E \ll V_0$ then this simplifies
- The transmission coefficient is exponentially dependent on the barrier width and α
  - The wider the barrier, the harder tunneling is
  - The larger the barrier height $V_0 - E$ or mass m, the harder tunneling is
- In general with a potential barrier for any shape we can break it up into potential steps and integrate

Examples of Quantum Tunneling

- Field emission: consider electrons in a piece of metal in a vacuum;