Lecture 25 (2-9), Nov 15, 2022

Copenhagen Interpretation Revisited

- The energy of a system is indeterminate until the measurement
  - Before the measurement, the wavefunction is a superposition of many eigenstates (e.g. $\psi_1, \psi_2, \psi_3, \cdots$)
- Measurements of the total energy will give the expectation value of $\hat{H}$
  - Each individual measurement will only give you $E_1, E_2, \cdots$ in quantized levels, but the probabilities of each energy makes it so that on average you measure the expectation value
- When the measurement happens, the wavefunction collapses to a certain eigenstate

ψ_n Form an Orthonormal Basis

- Orthonormality of ψ_n:
  - Define the inner product as $\int \psi_n^* \psi_m \, \mathrm{d}x$
  - Orthonormality means that $\int \psi_n^* \psi_m \, \mathrm{d}x = \delta_{mn}$ where δ_{mn} is the Kronecker delta
    - When $n = m$ since ψ is normalized this is trivially true
    - Consider the infinite square well, then $\int \psi_n^* \psi_m \, \mathrm{d}x = \frac{2}{a} \int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) \mathrm{d}x = \delta_{mn}$
- Completeness:
  - Any arbitrary function can be expressed as some linear combination of ψ_n
  - $f(x) = \sum_{n=1}^{\infty} c_n \psi_n$
  - For the infinite square well, this is $\sqrt{\frac{2}{a}} \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{a}\right)$
    - This is a Fourier series, so this proves completeness
- Using these, we can show $\int \psi_m^* f(x) \, \mathrm{d}x = \sum_{n=1}^{\infty} c_n \int \psi_m^* \psi_n \, \mathrm{d}x = \sum_{n=1}^{\infty} \delta_{mn} = c_m$

Interpretation of c_n

- $\langle \hat{H} \rangle = \sum_{n=1}^{\infty} |c_n|^2 E_n$
- $|c_n|^2$ is the "weight" of each energy