Lecture 16, Oct 18, 2022

Superposition of Waves

- Monochromatic waves have only one wavelength/frequency
- Waves can be superimposed
- Superimposing two waves creates beat phenomena
  - The wave is a multiplication of the two waves
  - The first wave has the average wave number and frequency
  - The second wave has $\frac{1}{2}(k_2 - k_1)$ and $\frac{1}{2}(\omega_2 - \omega_1)$
- The higher frequency makes the carrier frequency in the beat; the lower frequency makes the modulating frequency
  - The beat frequency is $\frac{1}{2}|\omega_2 - \omega_1|$
- This is used in AM radio

Important

Although the beat frequency has $\omega = \frac{1}{2}(\omega_2 - \omega_1)$, the frequency we hear is effectively twice this amount, at $\omega_2 - \omega_1$

Dispersion

- What happens when not al l the waves travel with the same frequency?
- In dispersive media, v and ω are functions of k, and wave speed depends on frequency
- A pulse, made of a superposition of waves, spreads out as it travels as the different components travel at different speeds
- The relation $\omega(k)$ is the *dispersion relation*
- This can be seen in e.g. prisms where in a dispersive medium, the refractive index for different wavelengths is different
- Phase velocity is defined as $v = \frac{\omega_0}{k_0}$, the ratio of the average frequency and wave number
  - The phase velocity is how fast a disturbance travels
  - The velocity of the carrier wave
- Group velocity is defined as $v_g = \frac{\Delta\omega}{\Delta k} = \frac{\omega(k_2) - \omega(k_1)}{k_2 - k_1}$
  - The group velocity is how fast the envelope travels
  - The velocity of the modulating wave
  - $v_g = \left.\frac{\mathrm{d}\omega}{\mathrm{d}k}\right|_{k=k_0}$
  - $v_g = v - \lambda\frac{\mathrm{d}v}{\mathrm{d}\lambda}$