Lecture 15, Oct 7, 2022

Energy and Power of a Travelling Wave

- To propagate a wave, we need a power source; we want to find this power
- Energy is carried by either kinetic or potential energy
- Power $P = Fv$ so $P(x,t) = -\tau\dfrac{\partial y}{\partial x}\dfrac{\partial y}{\partial t}$
  - $P(x,t) = \sqrt{\mu\tau}A^2\omega^2\sin^2(kx - \omega t + \phi_0)$
- Average power is $\dfrac{1}{2}\sqrt{\mu\tau}A^2\omega^2$
  - Max power is $\sqrt{\mu\tau}A^2\omega^2$
  - Note $Z = \sqrt{\mu\tau}$ and $A\omega = v_{max}$
- Sound waves propagate in multiple directions, so we define *intensity* $I = \dfrac{P_{avg}}{S}$, average power per unit area
  - Intensity is given by $\dfrac{\sqrt{\rho B}A^2\omega^2}{2} = \dfrac{\Delta p_{max}^2}{2\rho v} = \dfrac{\Delta p_{max}^2}{2\sqrt{\rho B}}$
  - In 2D, $I \propto \dfrac{1}{r}$
  - In 3D, $I \propto \dfrac{1}{r^2}$; if power at the source is P, then $I = \dfrac{P}{4\pi r^2}$

Attenuation

- Energy is lost to the wave medium as heat as the wave passes through
- Rate of absorption is proportional to wave intensity
- $\dfrac{\mathrm{d}I}{\mathrm{d}x} = -\alpha I$ where α is the *attenuation coefficient*
  - $I(x) = I(x_0)e^{-\alpha(x-x_0)}$
  - Higher frequency gives a higher attenuation
- Attenuation and spreading are additive
  - Multiply by $\left(\dfrac{r_0}{r}\right)^{N-1}$ to add the spreading, where N is the number of dimensions
  - $I(r) = I(r_0)e^{-a(r-r_0)}\left(\dfrac{r_0}{r}\right)^{N-1}$
- Note usually attenuation is given per unit length, but sometimes it's given per wavelength

Intensity Level

- Intensity level is measured in decibels, $\beta = (10\text{dB})\log\left(\dfrac{I}{I_0}\right)$
- For us, every $10\times$ increase in intensity sounds twice as loud
- Human range of hearing is from $I_0 = 1\times10^{-12}\,\text{W/m}^2$ (threshold of hearing) to $I = 1\times10^{1}\,\text{W/m}^2$ (threshold of pain)
- To calculate decibels of (sound) intensity, use $I_0 = 1\times10^{-12}\,\text{W/m}^2$ (remember to use a base 10 log!)