Lecture 2, Sep 9, 2022

Classification of Differential Equations

- Ordinary vs Partial Differential Equations
  - PDEs have partial derivatives, resulting from the presence of multiple independent variables
- Order
  - The highest derivative that appears in the equation
- Linear vs Nonlinear
  - The most general nth order ODE can be expressed as $F(t, y, y', \cdots, y^{(n)}) = 0$
  - A linear ODE can be written as $a_0(t)y^{(n)} + a_1(t)y^{(n-1)} + \cdots + a_n(t)y = g(t)$
    - a_n can depend on t and t alone
  - The linear DE is homogeneous if $g(t) = 0$
- Autonomous vs Nonautonomous
  - An autonomous ODE does not explicitly depend on t, e.g. $y' = y$ is autonomous, $y' = ty$ is not
- Separable vs Nonseparable
  - A first order ODE $\dfrac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$ is separable if we can decompose $f(t, y) = p(t)q(y)$
- Example: $\dfrac{\mathrm{d}u}{\mathrm{d}t} = -k(u - T_0)$ is a first order, linear, nonhomogeneous, autonomous, separable ODE

Lotka-Volterra (Predator-Prey)

- Modelling the number of zombies in an apocalypse, where x is the number of people and y is the number of zombies, assumptions:
  1. Zombies eat people
     - $x' = -\beta xy$
     - The rate at which people get eaten is proportional to the number of zombies and people
  2. People reproduce
     - $x' = \alpha x$
  3. Zombies suffer natural death and emigration
     - $y' = \delta xy - \gamma y$
     - Zombies flourish when they're being fed; the more there are, the more are dying of natural causes
- This is summarized in the system: $\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \alpha x - \beta xy \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -\gamma y + \delta xy \end{cases}$

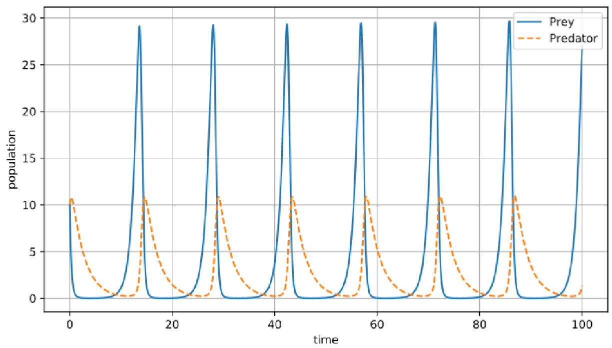

Figure 1: Cycle of predator-prey population