Lecture 14, Oct 7, 2022

Euler's Method

- Iterative method: Solve $y' = f(t, y)$ by $y_{n+1} = y_n + hf(t_n, y_n)$

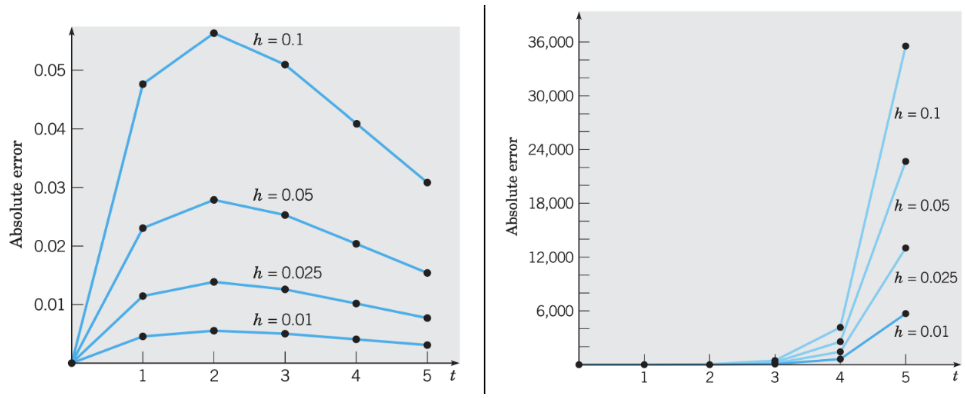

Figure 1: Comparison of errors in Euler's method for $y' = \frac{3}{2} - t - \frac{1}{2}$ and $y' = 4 - t + 2y$

- A larger step size always increases the error in Euler's method
- Error accumulates in Euler's method, but this is not always the case – sometimes the absolute error can decrease
  - Why do approximations work better sometimes?

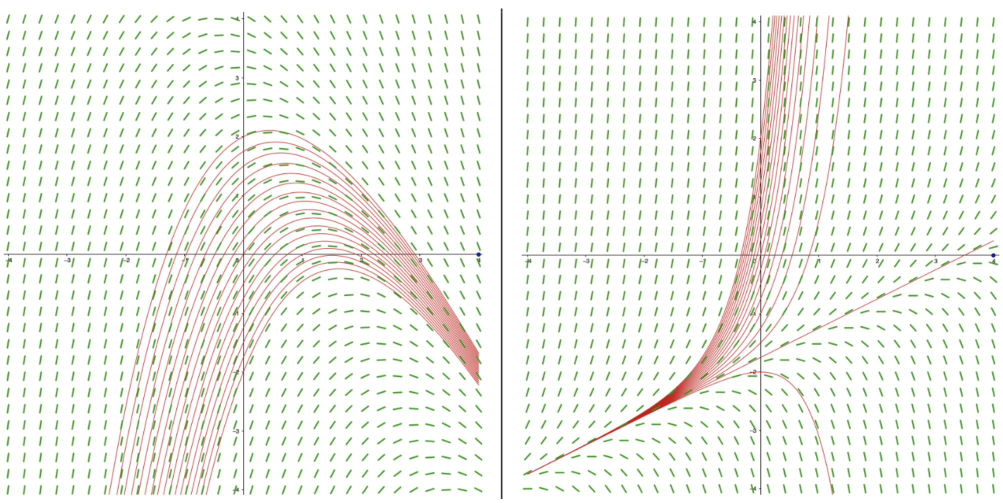

Figure 2: Comparison of solutions for $y' = \frac{3}{2} - t - \frac{1}{2}$ and $y' = 4 - t + 2y$

- Notice the left has a solution of $y(t) = 7 - 2t - Ce^{-\frac{t}{2}}$ and the right has a solution of $y(t) = -\frac{7}{4} + \frac{1}{2}t + Ce^{2t}$
  - In the left one, all solutions converge towards $y(t) = 7 - 2t$, while in the right solution diverge
  - A small error in the left eventually decays away, while a small error in the right blows up