Lecture 13, Oct 6, 2022

Repeated Eigenvalues Examples

- Example: $\boldsymbol{x}' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \boldsymbol{x}, \lambda_1 = \lambda_2 = 2, \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
  - Generalized eigenvector: $(\boldsymbol{A} - \lambda \boldsymbol{I})\boldsymbol{w} = \boldsymbol{v} \implies \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \boldsymbol{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \implies \boldsymbol{w} = \begin{bmatrix} k \\ -1 - k \end{bmatrix}$
  - General solution: $\boldsymbol{x} = c_1 e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 \left(te^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + e^{2t} \begin{bmatrix} k \\ -1 - k \end{bmatrix} \right)$
    * Notice that the term with $\boldsymbol{w}$ is $ke^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + e^{2t} \begin{bmatrix} 0 \\ -1 \end{bmatrix}$; the former can be absorbed into the c_1 term
    * We can also just choose k to be whatever we want
  - Simplified, $\boldsymbol{x} = c_1 e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 \left(te^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + e^{2t} \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right)$
  - As $t \to \infty$ the solution is dominated by the te^{2t} term or the vector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
  - As $t \to -\infty$ the solution goes to 0, but is still dominated by te^{2t}
  - The equilibrium at 0 is unstable; it is an *improper* equilibrium (also known as an improper node as all solutions emerge from it)