Lecture 1, Sep 8, 2022

- Modelling temperature of a boba cup on a hot day: $u(t)$ where T_0 is the surrounding temperature
  - u is the dependent variable, t is the independent variable
- What is the problem with the following models?
  - $u' = u^2$
    * Temperature increases forever
  - $u' = u'' + 2u$
    * No dependence on the surrounding temperature
  - $u' = u - T_0$
    * u does not approach T_0
  - $u' = T_0 - u$
    * The environment is not taken into account (e.g. if the type of liquid changed, the equation can't account for it)
- Newton's Law of Cooling: The rate of change of temperature is negatively proportional to the difference between the temperature difference between the object and its surroundings
  - $u' = -k(u - T_0)$
    * k is the transmission coefficient

Note

Newton was an avid boba drinker[citation needed]

- Solution:
  - $$\begin{aligned} & \frac{\mathrm{d}u}{\mathrm{d}t} = -k(u - T_0) \\ \implies & \frac{\frac{\mathrm{d}u}{\mathrm{d}t}}{u - T_0} = -k \\ \implies & \frac{\mathrm{d}}{\mathrm{d}t} \ln\lvert u - T_0 \rvert = -k \\ \implies & \ln\lvert u - T_0 \rvert = -kt + C \\ \implies & u - T_0 = Ae^{-kt} \\ \implies & u = Ae^{-kt} + T_0 \end{aligned}$$
  - This gives us a family of curves, all with different initial conditions (*integral curves*)
  - In general we know $u(0)$ or $u(t_0)$ for some t_0 so we can solve for A