Lecture 6, Sep 20, 2022

Functional Completeness of NAND and NOR

- DeMorgan's theorem allows us to implement any SOP circuit can be implemented using only NAND gates:
  - Example: $f = x_1x_2 + x_2x_3$
    $$\begin{aligned} &= \overline{\overline{(x_1x_2)}} + \overline{\overline{(x_2x_3)}} \\ &= \overline{\overline{x_1x_2} \cdot \overline{x_2x_3}} \end{aligned}$$
  - For a POS circuit, convert to SOP first
- Any POS circuit can be implemented using NOR gates:
  - Example: $f = (x_1 + x_2)(x_2 + x_3)$
    $$\begin{aligned} &= \overline{\overline{x_1 + x_2}} \cdot \overline{\overline{x_2 + x_3}} \\ &= \overline{\overline{x_1 + x_2} + \overline{x_2 + x_3}} \end{aligned}$$
  - Likely, for a SOP circuit, convert to POS first

Example

- Gumball factory
- s_2 normally 0, but 1 if gumball is too large
- s_1 normally 0, but 1 if too small
- s_0 normally 0, but 1 if too light
- Desired behaviour: $f = 1$ when gumball is either (too large) or (too small and too light)
  - By inspection, $f = s_2 + s_1s_0$
- Truth table:

$s_2s_1s_0$	f
000	0
001	0
010	0
011	1
100	1
101	1
110	1
111	1

- Minterms are the last 5 rows:
  - $f = \bar{s}_1s_1s_0 + s_2\bar{s}_1\bar{s}_0 + s_2\bar{s}_1s_0 + s_2s_1\bar{s}_0 + s_2s_1s_0$
- Simplify: $f = \bar{s}_1s_1s_0 + s_2\bar{s}_1\bar{s}_0 + s_2\bar{s}_1s_0 + s_2s_1\bar{s}_0 + s_2s_1s_0$
  $$\begin{aligned} &= s_1s_0(\bar{s}_2 + s_2) + s_2\bar{s}_1(s_0 + \bar{s}_0) + s_2\bar{s}_0(\bar{s}_1 + s_0) && \text{Rule 7 + Distributivity} \\ &= s_1s_0 + s_2\bar{s}_1 + s_2\bar{s}_0 && \text{Combination} \\ &= s_1s_0 + s_2(\bar{s}_1 + \bar{s}_0) && \text{Distributivity} \\ &= s_1s_0 + s_2\overline{s_1s_0} && \text{DeMorgan's Theorem} \\ &= s_1s_0 + s_2 && \text{Rule 16} \end{aligned}$$

Example 2

- Derive a minimal POS expression for $f(x_1, x_2, x_3) = \prod M(0, 2, 4)$

$x_1x_2x_3$	f	$\bar{f}$
000	0	1
001	1	0
010	0	1
011	1	0
100	0	1
101	1	0
110	1	0
111	1	0

- $f = (x_1 + x_2 + x_3)(x_1\bar{x}_2 + x_3)(\bar{x}_1 + x_2 + x_3)$
  $= (x_1 + x_3)(x_2 + x_3)$ Combination (dual)
- Using the min terms of $\bar{f}$:
  - $\bar{f} = \bar{x}_1\bar{x}_2\bar{x}_3 + \bar{x}_1x_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3$
    $= \bar{x}_1\bar{x}_3 + \bar{x}_2\bar{x}_3$
  - Using DeMorgan's rule: $f = \bar{\bar{f}} = \overline{\bar{x}_1\bar{x}_3 + \bar{x}_2\bar{x}_3} = \overline{\bar{x}_1\bar{x}_2} \cdot \overline{\bar{x}_2\bar{x}_3} = (x_1 + x_2)(x_2 + x_3)$