Lecture 4, Sep 15, 2022

Logic Expressions: Sum of Products and Products of Sums

- Terminology:
  - Literal: any variable or its complement, e.g. $x, y, \bar{x}$
  - Product term: an AND operation (since AND is denoted with $\cdot$)
  - Sum term: an OR operation (since OR is denoted with $+$)
- SOP and POS are a way to convert any arbitrary truth table to a logic expression

Sum of Products

Definition

Sum of products: An expression written as an OR operation of AND operations, e.g. $xy + \bar{x}\bar{y}$

- Minterm: A product term that evaluates to 1 for exactly one row of a truth table
  - Given a truth table, the min term is formed by including x_i if $x_i = 1$, or $\bar{x}_i$ if $x_i = 0$
- SOP specifies the truth table based on its ones
- Canonical SOP (Sum-of-Products): SOP expression for a function that comprises its minterms
  - Canonical SOPs are not simplified
- Example:

x	y	z	minterm
0	0	0	$\bar{x}\bar{y}\bar{z}$
0	0	1	$\bar{x}\bar{y}z$
0	1	0	$\bar{x}y\bar{z}$
0	1	1	$\bar{x}yz$
1	0	0	$x\bar{y}\bar{z}$
1	0	1	$x\bar{y}z$
1	1	0	$xy\bar{z}$
1	1	1	xyz

- Example: function comprised of minterms $f(x, y, z) = \sum m(0, 1, 2, 3, 6, 7)$
  - Canonical SOP: $f(x, y, z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + xy\bar{z} + xyz$

Product of Sums

Definition

Product of sums: An expression written as an AND operation of OR operations

- Maxterm: A sum term that evaluates to 0 for exactly one row of a truth table
  - Given a truth table, include x_i if $x_i = 0$ in that row, else include $\bar{x}_i$
- POS specifies the truth table based on its zeroes
- Canonical POS: POS expression for a function that comprises its maxterms
- Example:

x	y	z	maxterm
0	0	0	$x + y + z$
0	0	1	$x + y + \bar{z}$
0	1	0	$x + \bar{y} + z$
0	1	1	$x + \bar{y} + \bar{z}$

x	y	z	maxterm
1	0	0	$\bar{x} + y + z$
1	0	1	$\bar{x} + y + \bar{z}$
1	1	0	$\bar{x} + \bar{y} + z$
1	1	1	$\bar{x} + \bar{y} + \bar{z}$

- Example: $f(x, y, z) = \sum m(0, 1, 6, 7) = \prod M(2, 3, 4, 5)$
  - Canonical POS: $f(x, y, z) = (x + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(\bar{x} + y + \bar{z})$
- For any truth table, we can use its 1s to derive the SOP, or the 0s to derive the POS
- Example:

x	y	f
0	0	0
0	1	1
1	0	1
1	1	1

- Equivalent representations:
  - POS: $f = (x + y)$
  - SOP: $f = \bar{x}y + x\bar{y} + xy$
- Generally if you have fewer 0s, use POS, if you have fewer 1s, use SOP